

UNC 2008



# Dynamics of Bridges Under Moving Loads

(Past, Present and Future)

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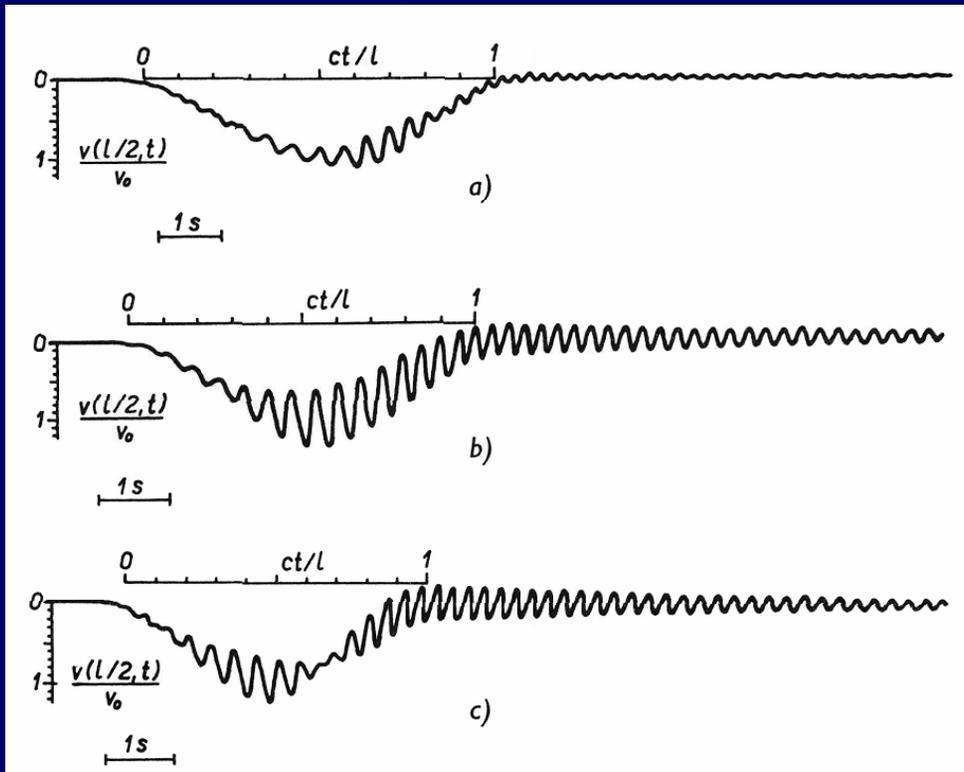
1. Introduction
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# 1. Introduction

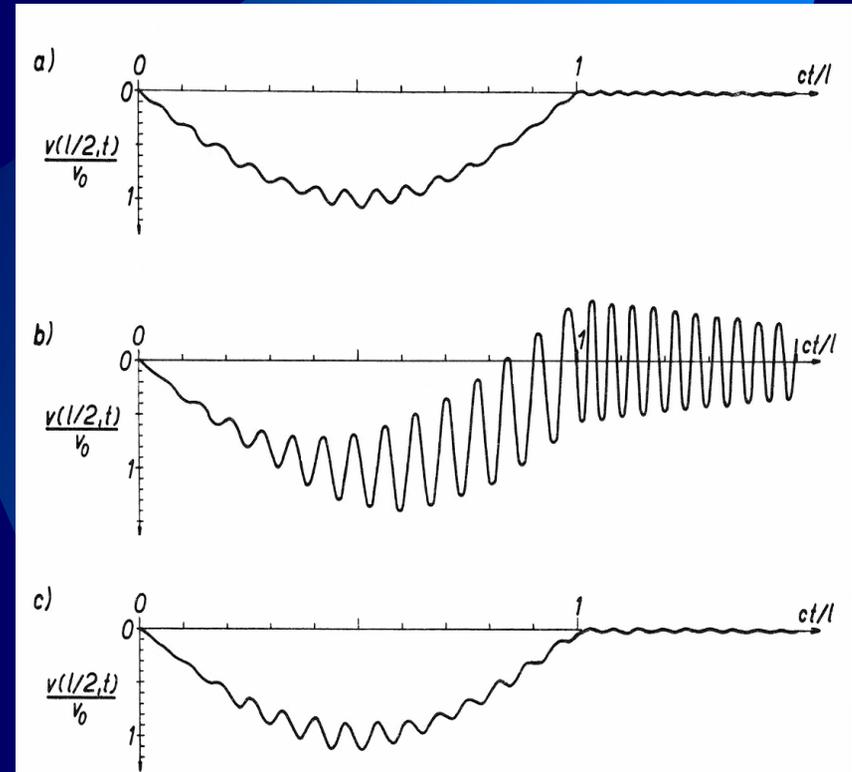
- First railway bridges in England
- First experimental and theoretical papers by Stokes (1849) and Willis (1849)
- Important progress by Timoshenko, Inglis and Koloušek

steam locomotives  $F(t) = F_0 \sin \Omega t$

# Steel bridge, $l = 56.56$ m



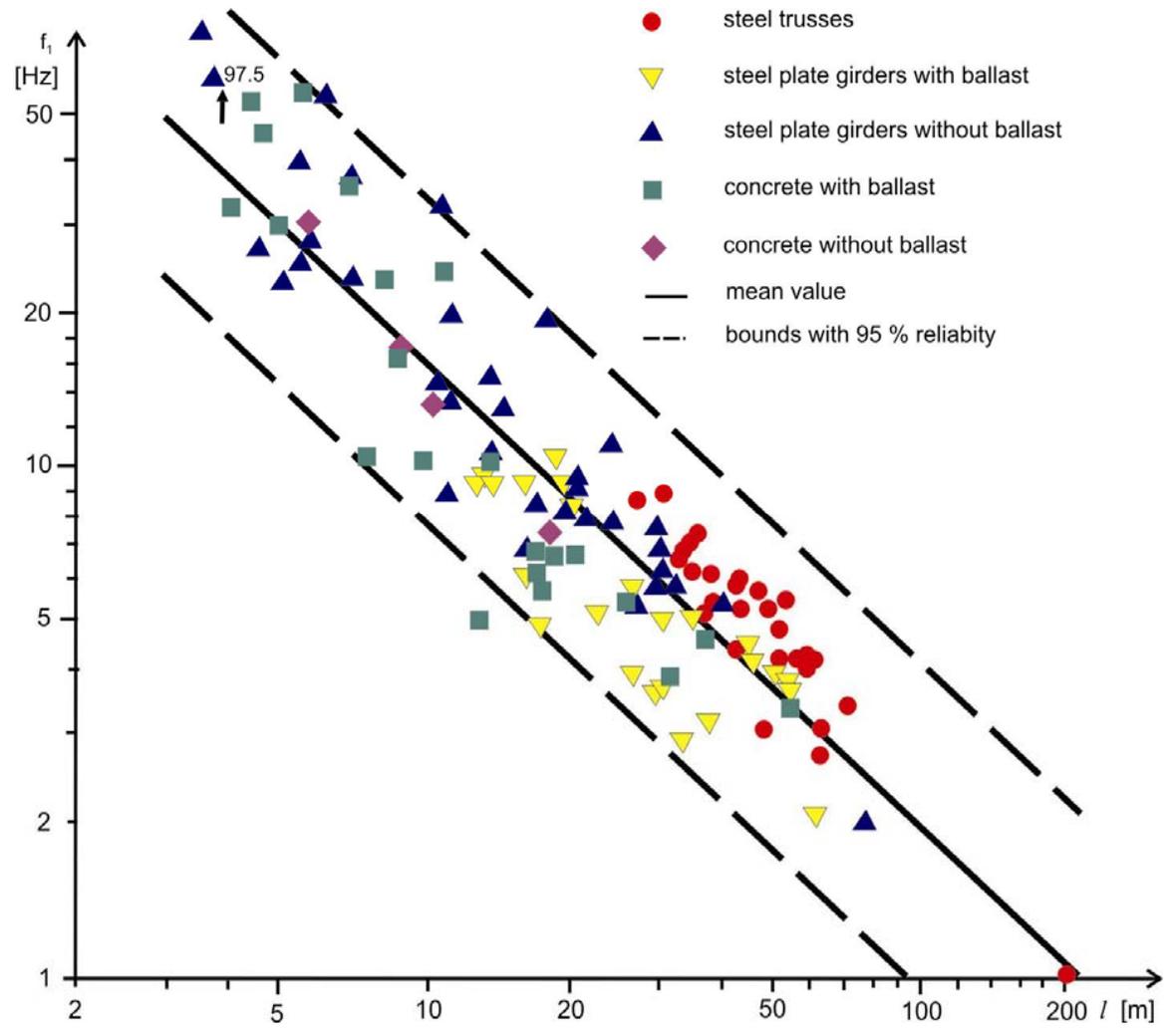
experiments



theory

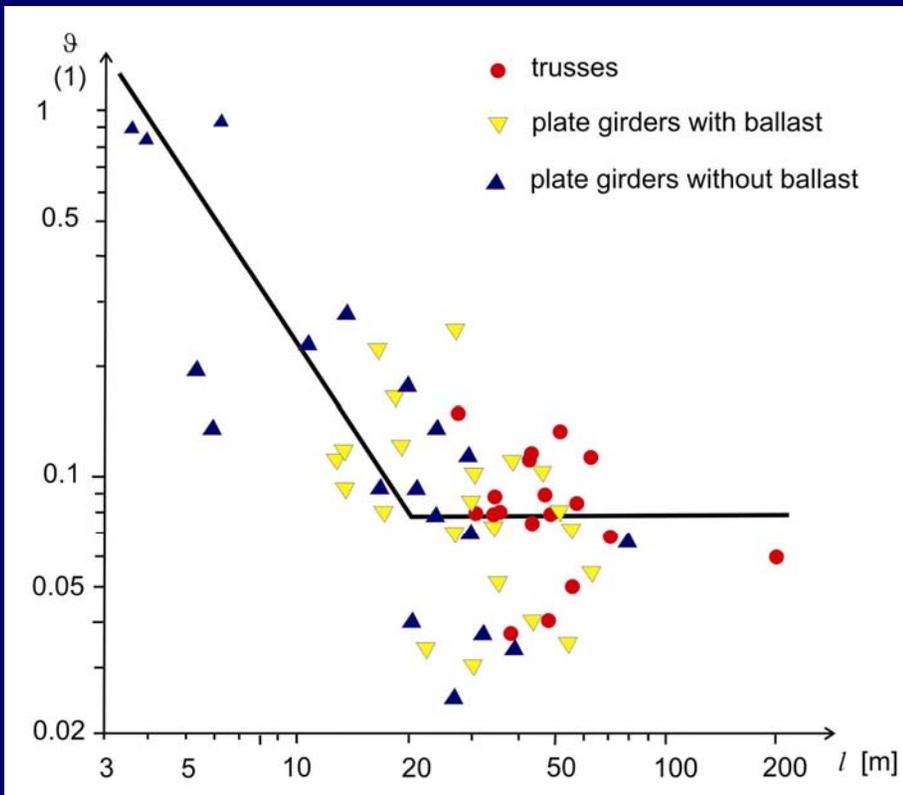
## 2. Past

- International investigations by ORE, ERRI and OSŽD
- Dynamic characteristics of bridges

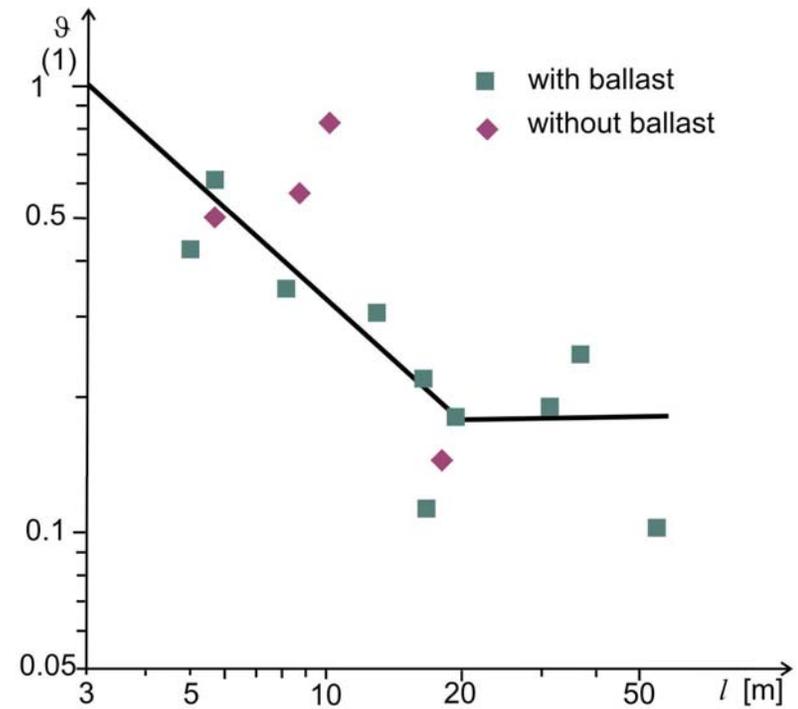


First natural frequencies of bridges

# Logarithmic decrements of damping

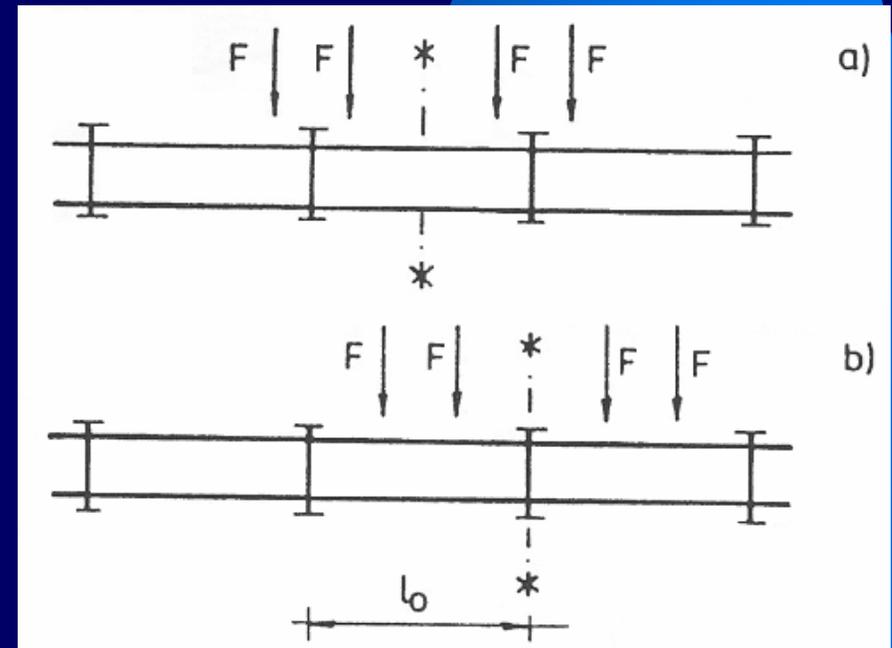
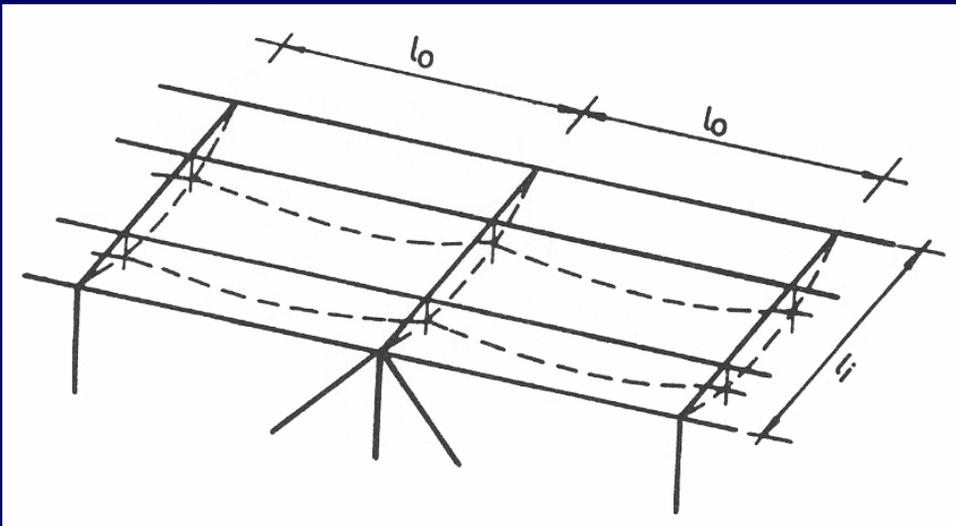


steel

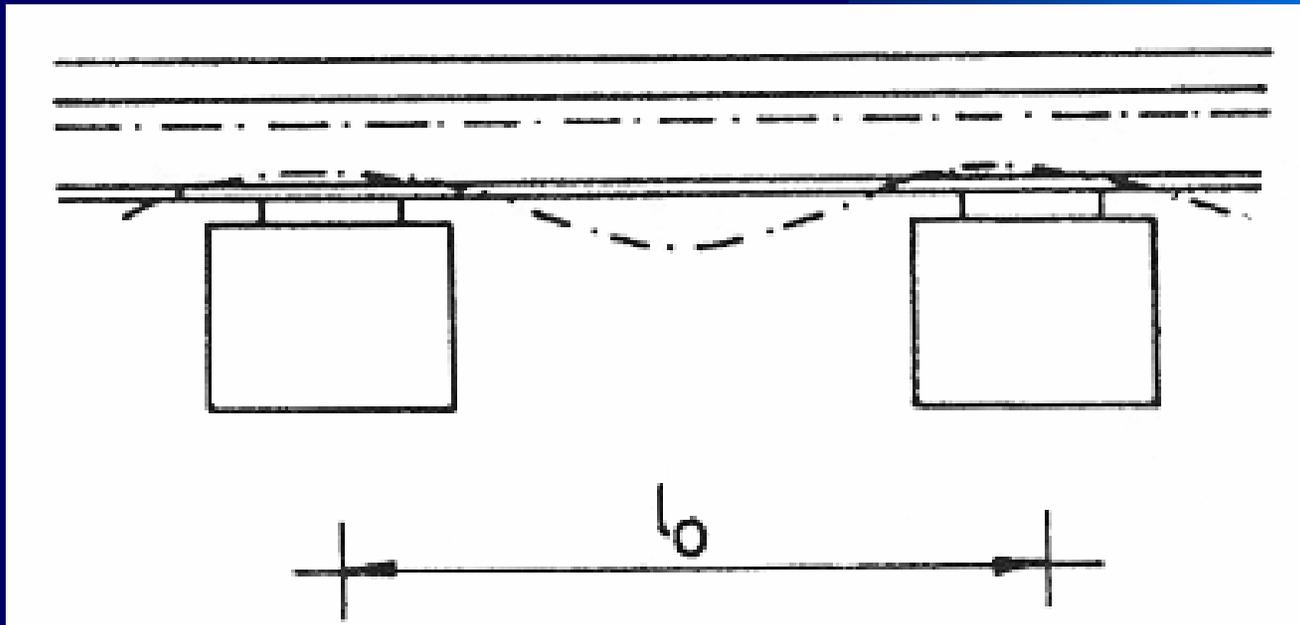


concrete

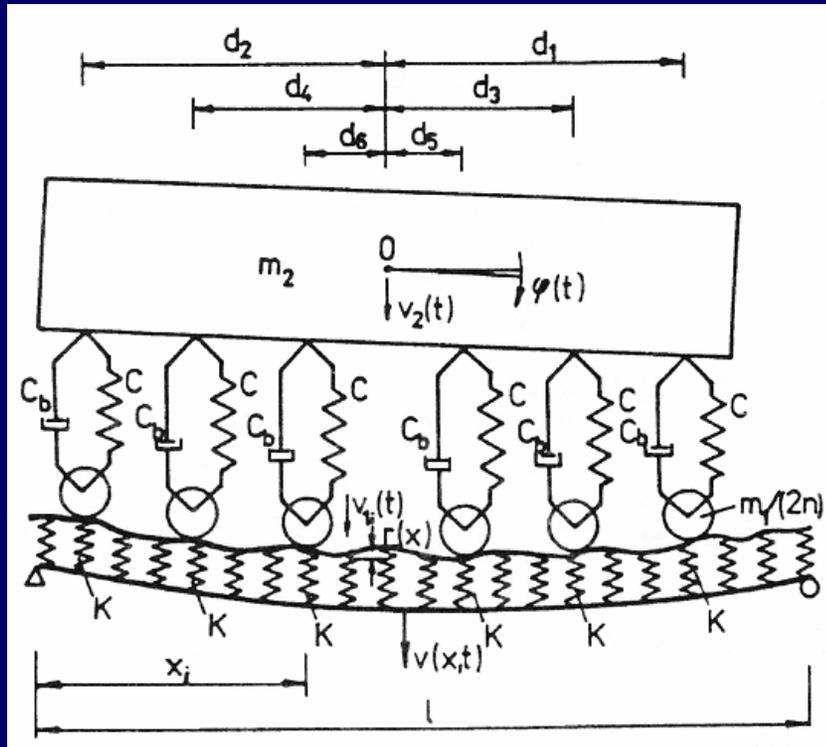
# Cross girder effect



# Sleeper effect



# Theoretical model



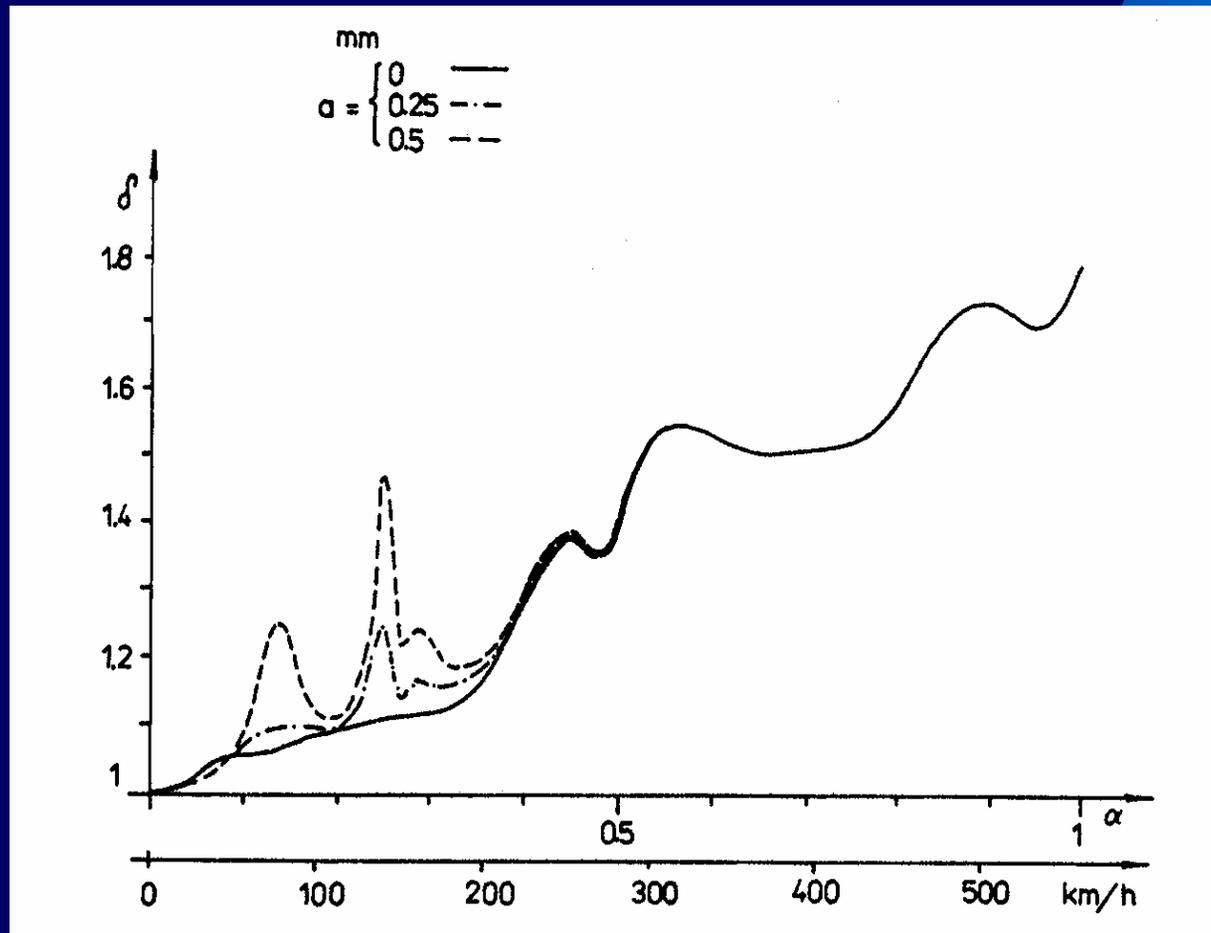
$$-I \frac{d^2 \bar{\varphi}(t)}{dt^2} + \sum_{i=1}^2 (-1)^i D_i [Z_i(t) + Z_{bi}(t)] = 0,$$

$$-m_3 \frac{d^2 v_3(t)}{dt^2} - \sum_{i=1}^2 [Z_i(t) + Z_{bi}(t)] = 0$$

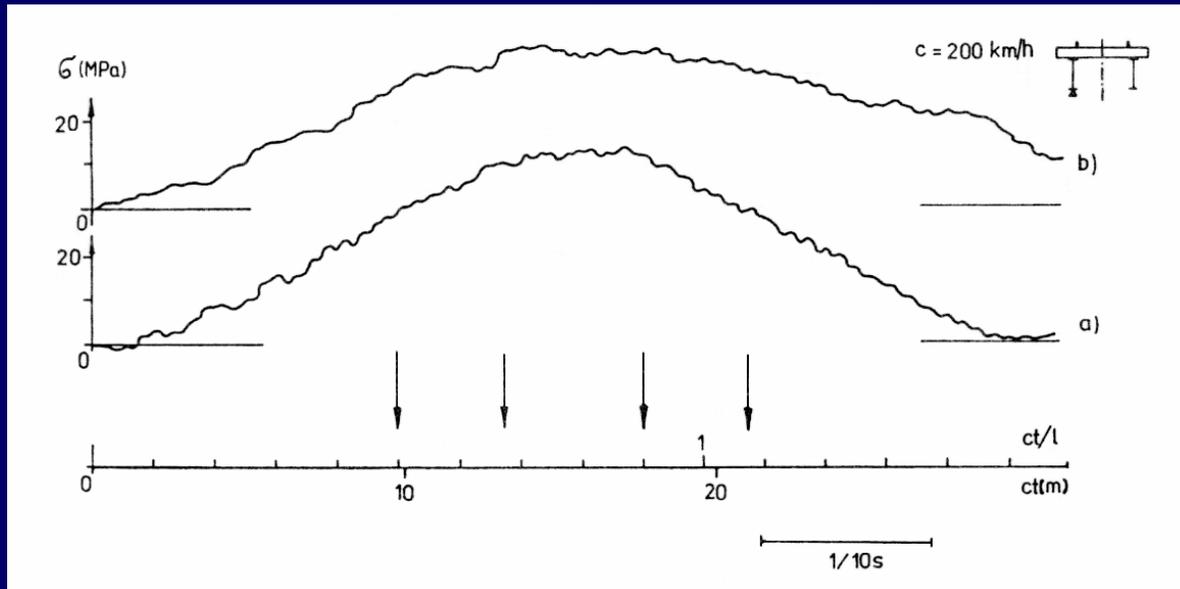
$$P_i + P_{3i} - m_i \frac{d^2 v_i(t)}{dt^2} + Z_i(t) + Z_{bi}(t) - \bar{R}_i(t) = 0; \quad i = 1, 2,$$

$$EJ \frac{\partial^4 v(x,t)}{\partial x^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + 2\mu\omega_b \frac{\partial v(x,t)}{\partial t} = \sum_{i=1}^2 \bar{\varepsilon}_i \bar{\delta}(x - x_i) \bar{R}_i(t).$$

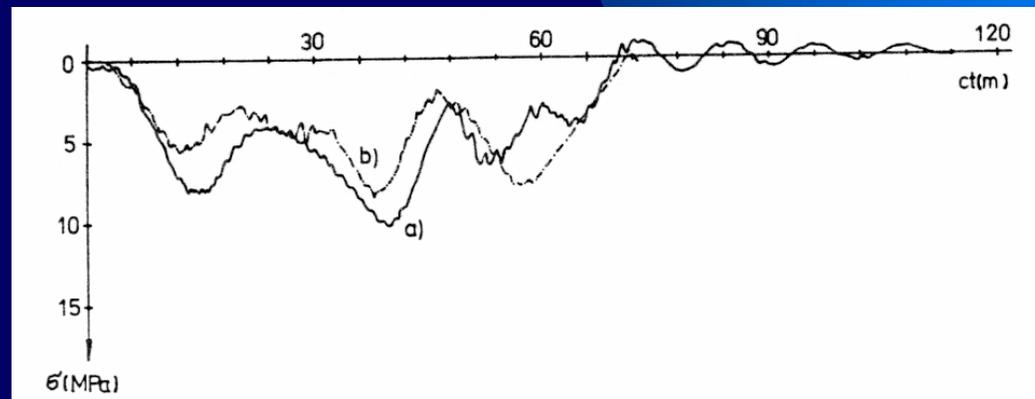
# Effect of the speed



# ORE experiments (DB, SNCF)



DB steel bridge,  $l = 19.6 \text{ m}$ ,  $200 \text{ km/h}$

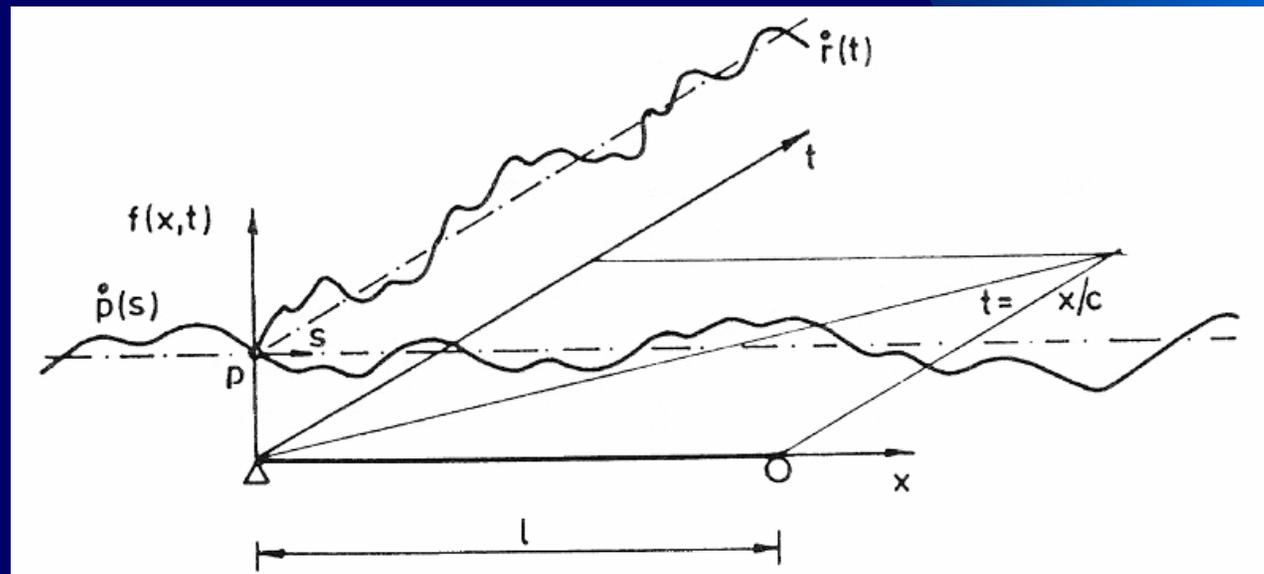
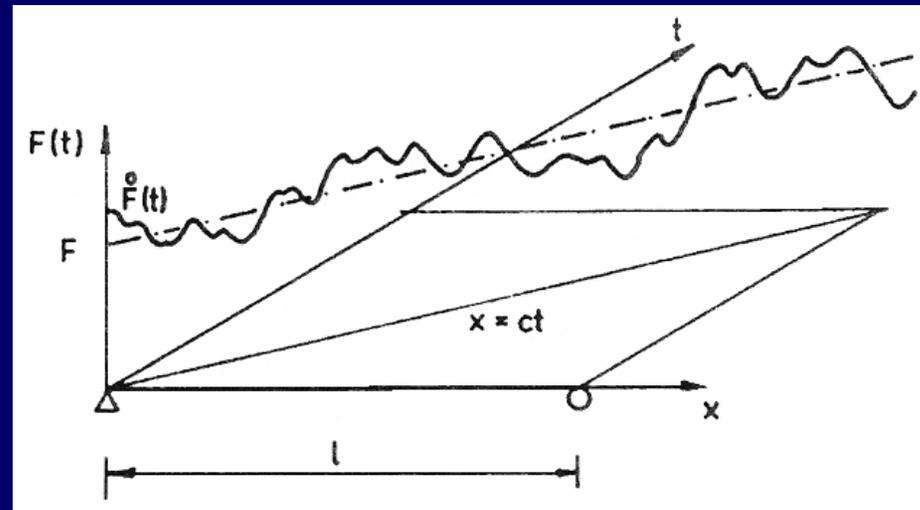


SNCF composite bridge,  $l = 26.4 \text{ m}$ ,  $241 \text{ km/h}$

# Stochastic concept

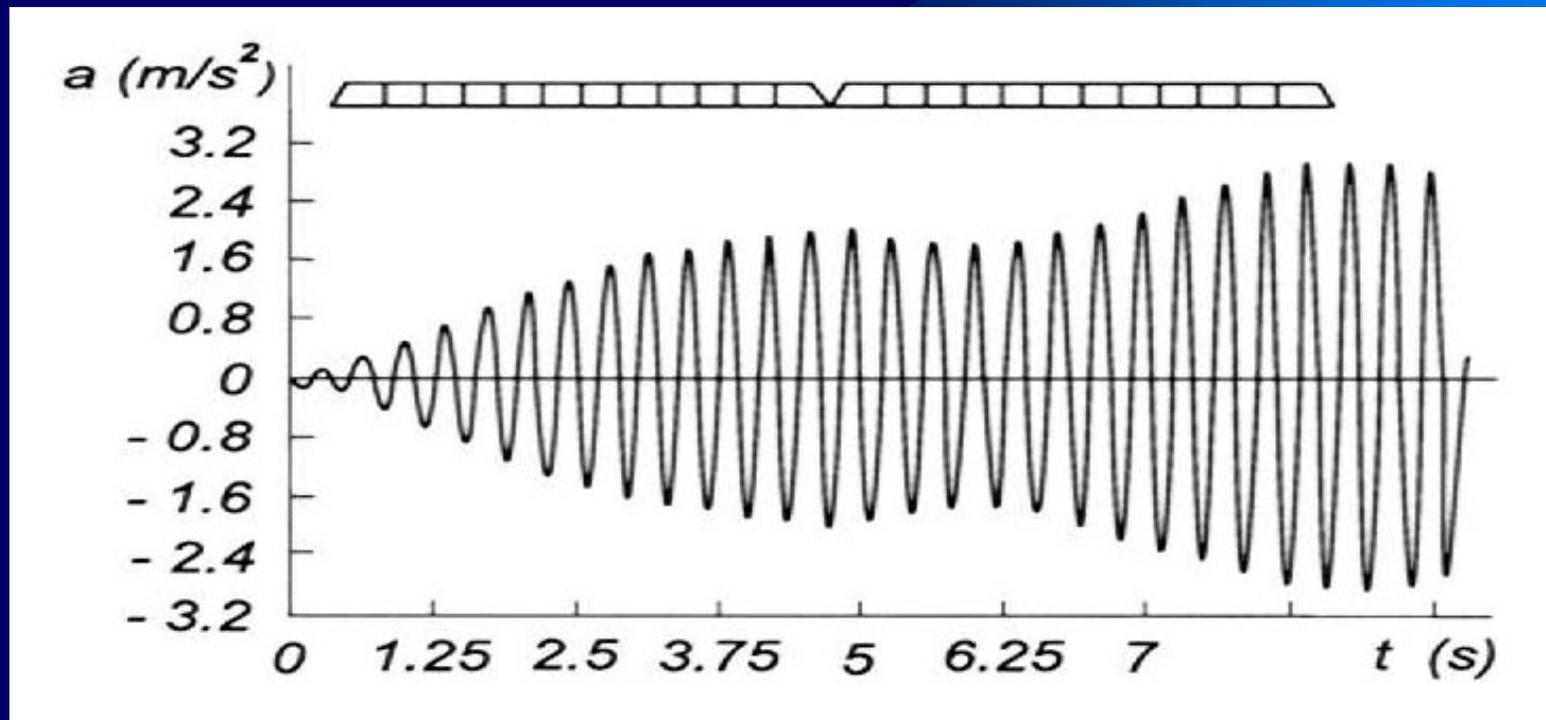
$$F(t) = F + \varepsilon \dot{F}(t),$$

$$f(x,t) = [p + \varepsilon \dot{p}(s)][1 + \dot{r}(t)]$$



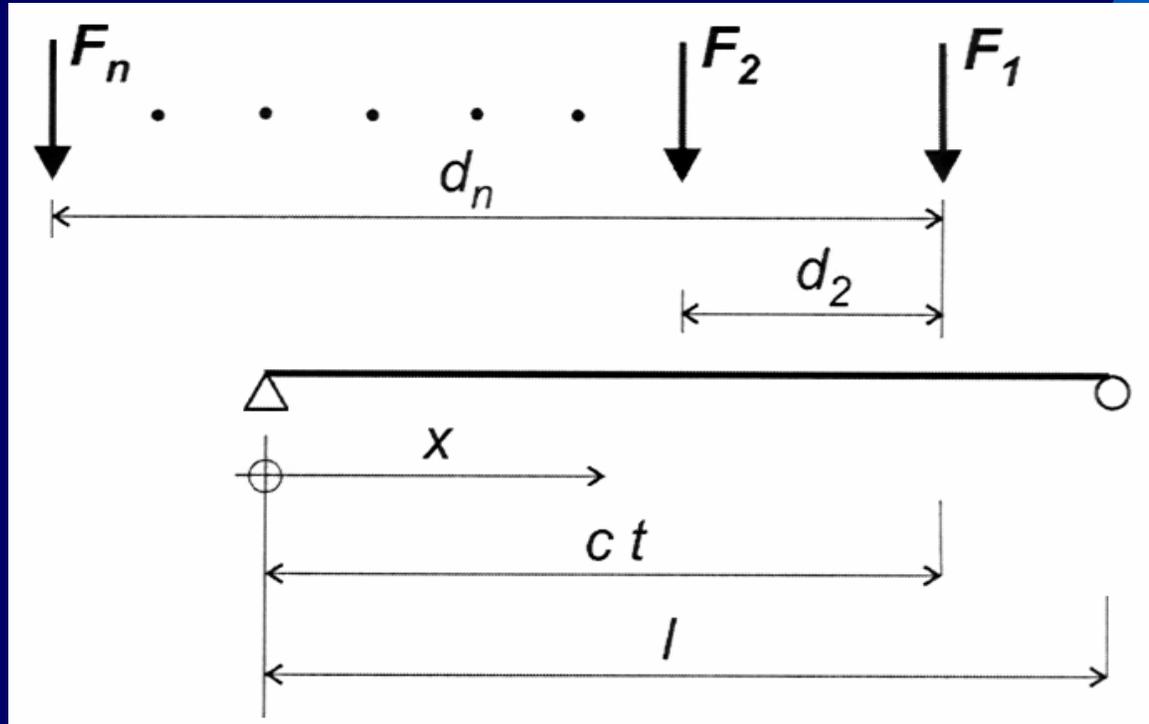
# 3. Present

## Resonant vibration

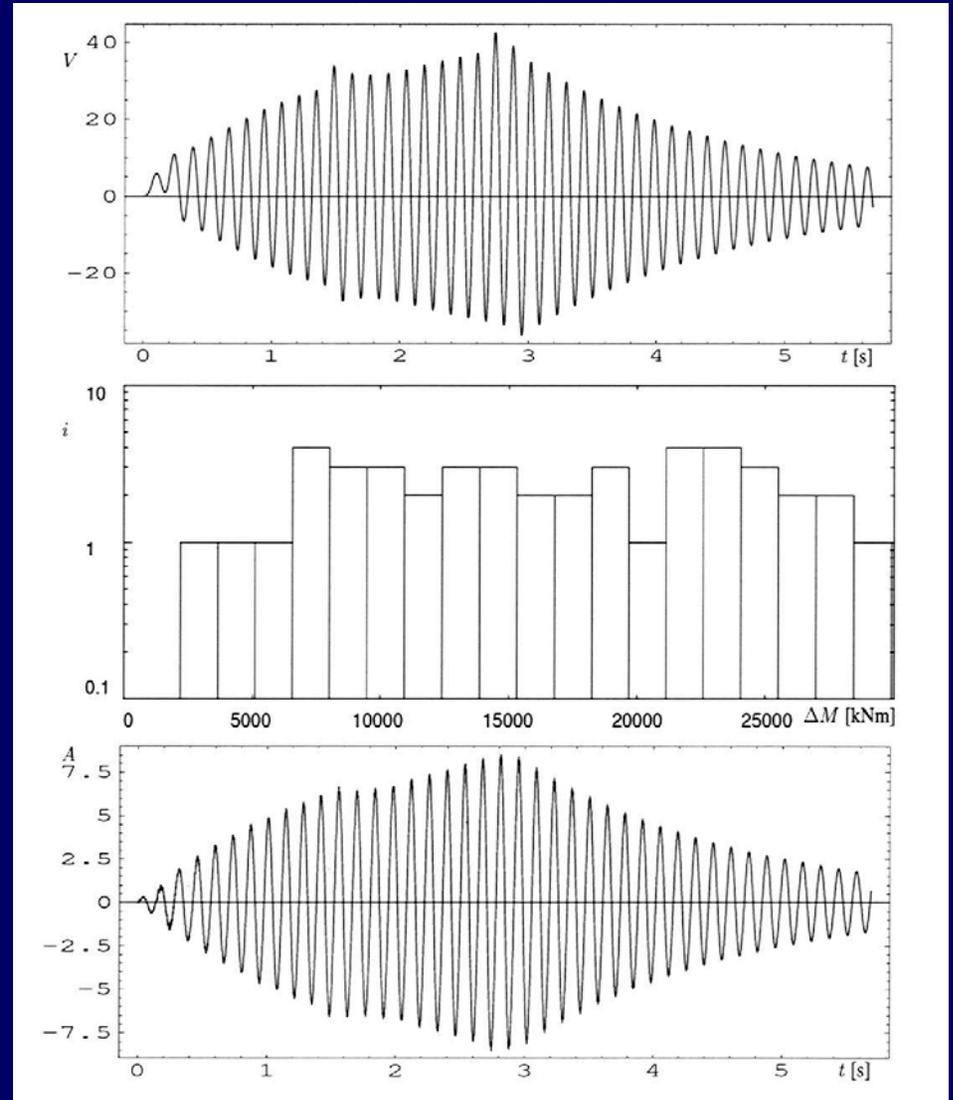
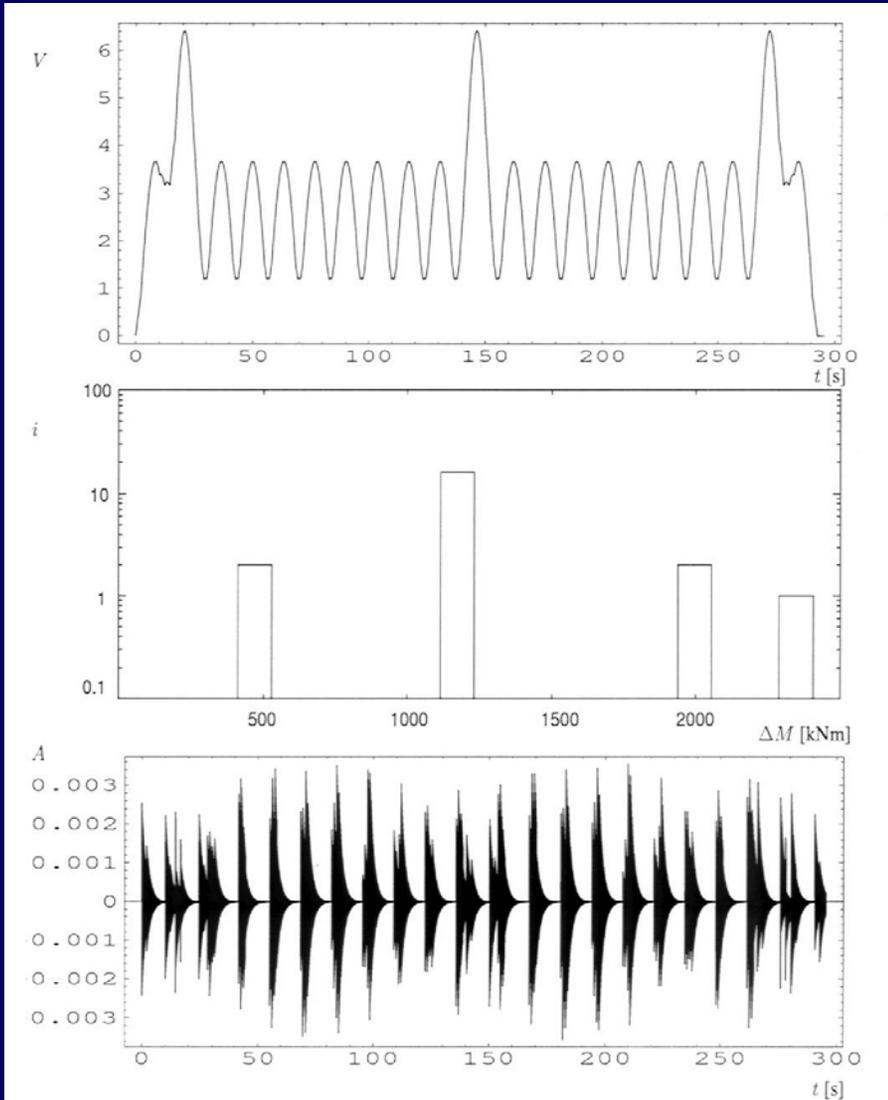


SNCF bridge,  $l = 38$  m, TGV, 192 km/h

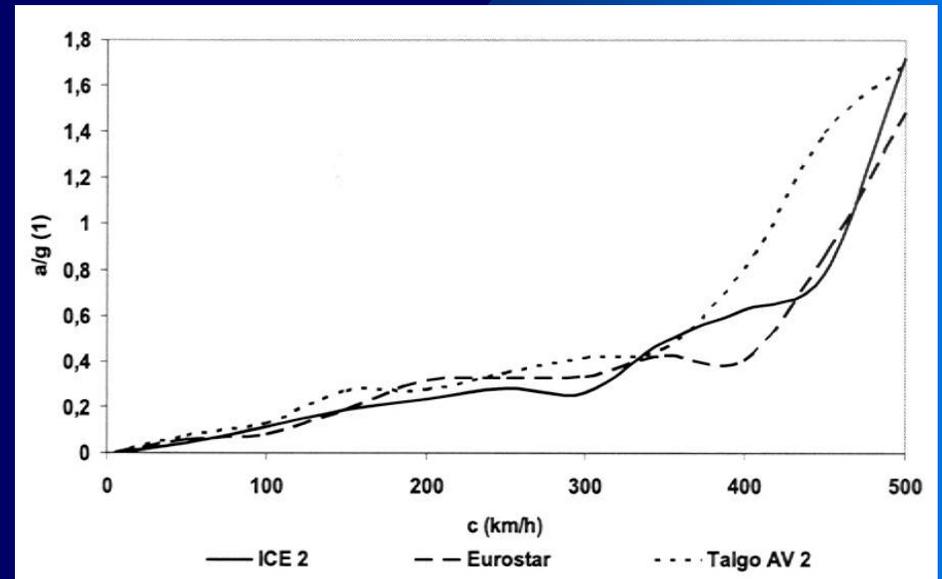
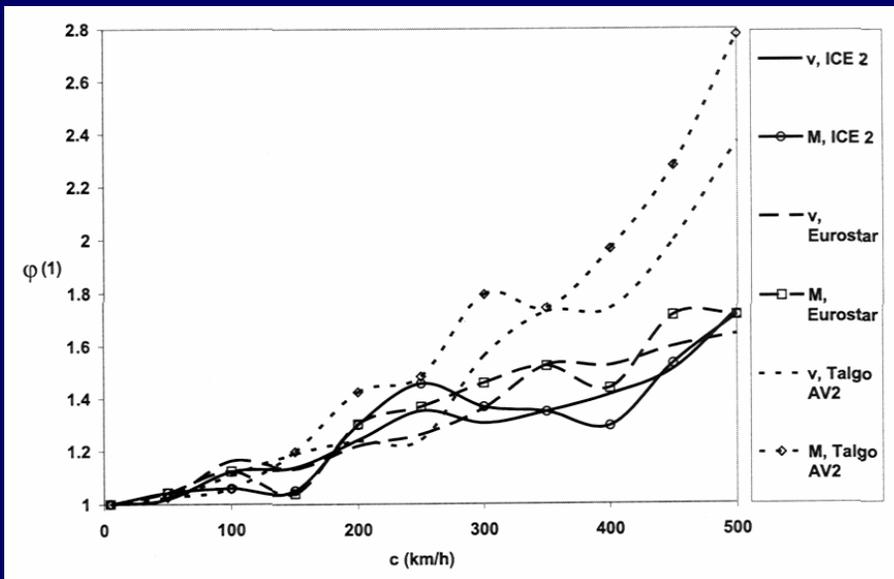
# Theoretical idealization



$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + 2\mu\omega_d \frac{\partial v(x,t)}{\partial t} = \sum_{n=1}^N \varepsilon_n(t) \delta(x - x_n) F_n$$



Vibration of a steel bridge at low and resonant speed



Effect of the speed on deflection and bending moments

steel bridge,  $l = 5 \text{ m}$

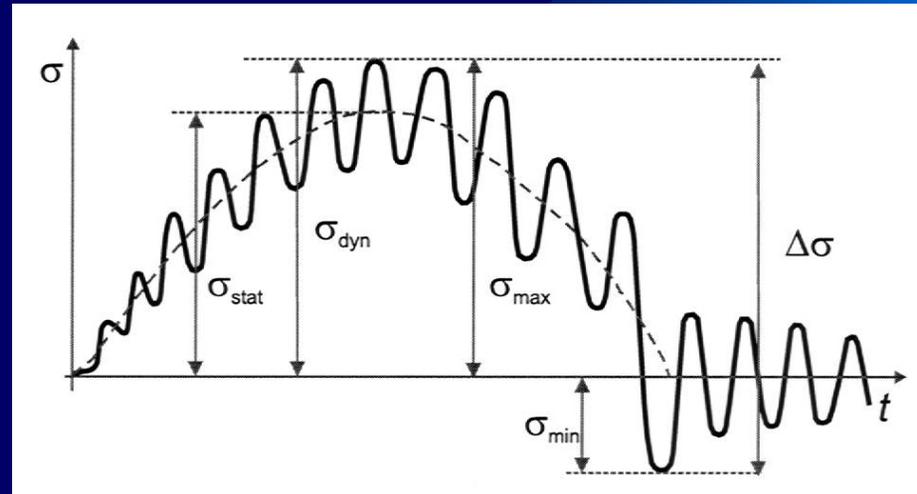
Effect of the speed on vertical acceleration

concrete bridge,  $l = 10 \text{ m}$

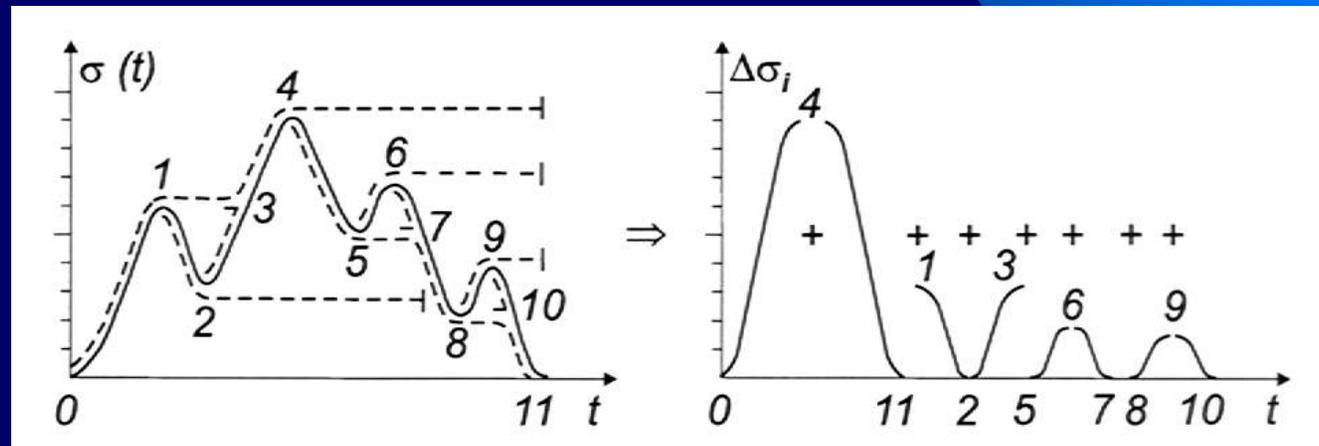
# Stress spectra

stress range

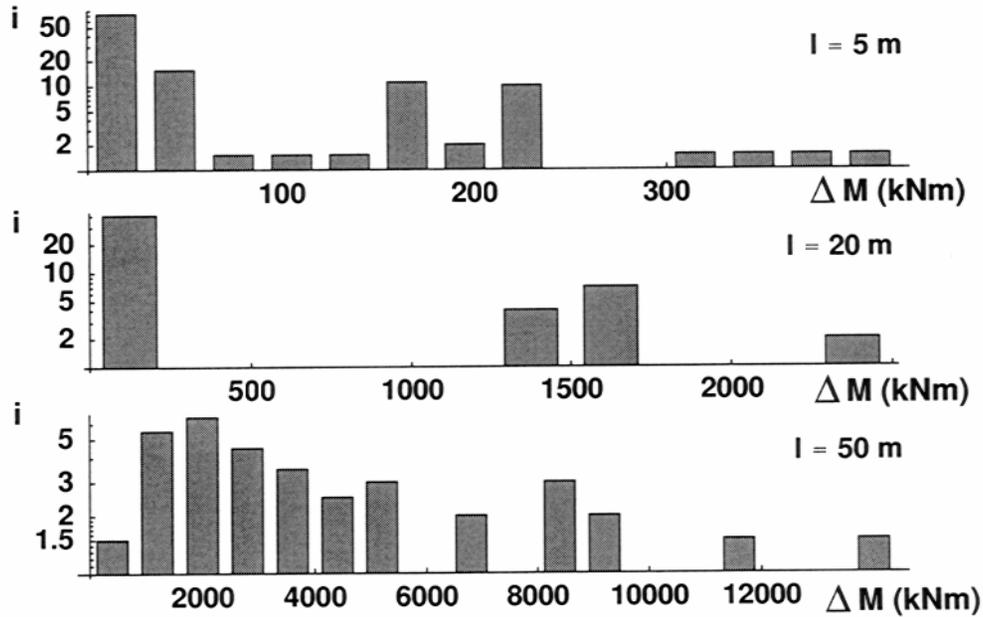
$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$



rain-flow

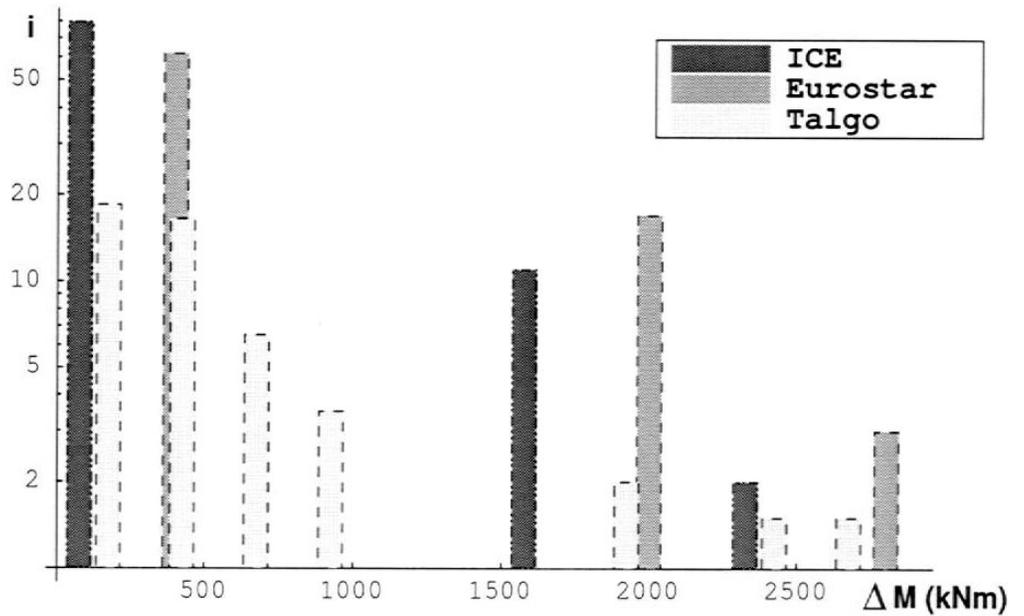


STEEL bridge, ICE, c=350 km h



# Stress spectra for steel and concrete bridges

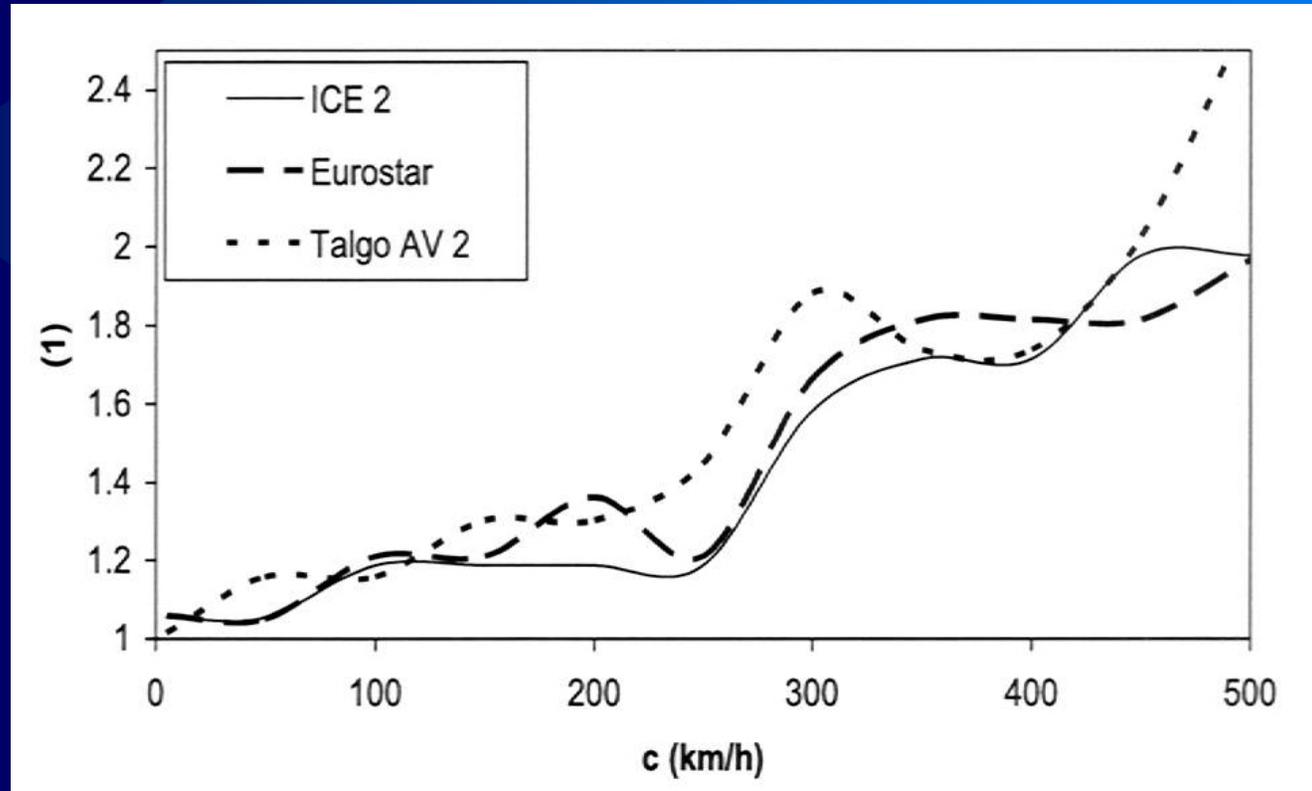
CONCRETE bridge, l=20m, c = 350 km h



# Stress ranges

## Effect of the speed

concrete bridge,  $l = 5$  m



# Critical speeds

$$c_{cr} = \frac{df_j}{k}, \quad j = 1, 2, 3, \dots, \quad k = 1, 2, 3, \dots, 1/2, 1/3, 1/4, \dots$$

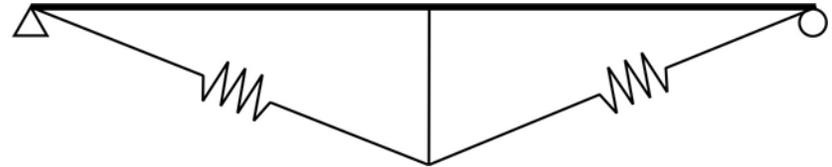
$$c_{cr} = \frac{2lf_j}{j}, \quad j = 1, 2, 3, \dots$$

# 4. Future

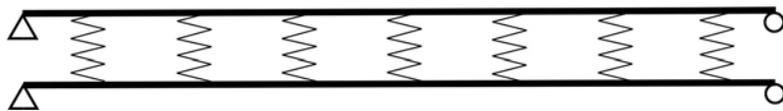
Elastic supports



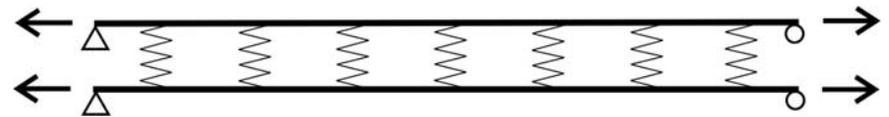
Triangular falsework



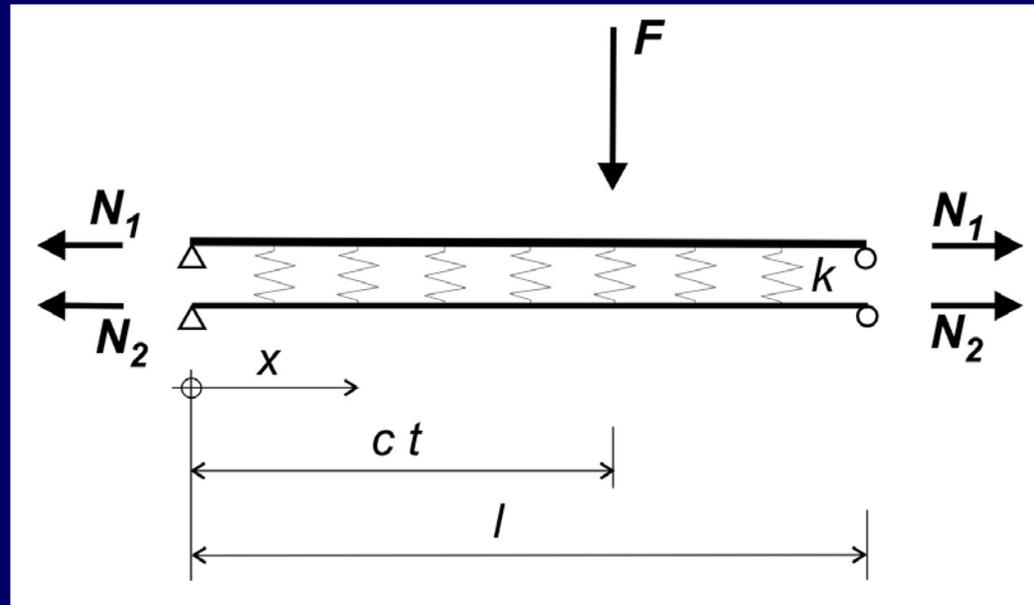
Double beam



Double string



## Beam coupled with a string



$$EI \frac{\partial^4 v_1(x, t)}{\partial x^4} - N_1 \frac{\partial^2 v_1(x, t)}{\partial x^2} + \mu_1 \frac{\partial^2 v_1(x, t)}{\partial t^2} + k [v_1(x, t) - v_2(x, t)] = \varepsilon(t) \delta(x - ct) F$$

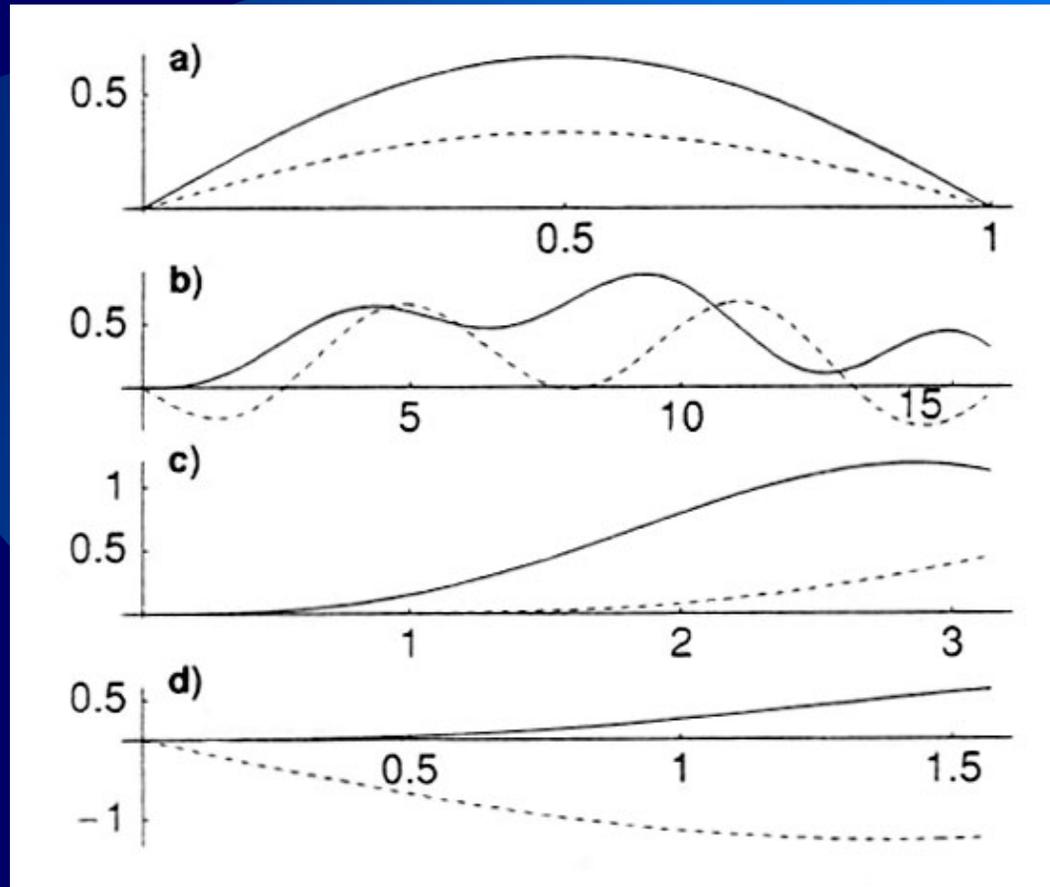
$$-N_2 \frac{\partial^2 v_2(x, t)}{\partial x^2} + \mu_2 \frac{\partial^2 v_2(x, t)}{\partial t^2} + k [v_2(x, t) - v_1(x, t)] = 0$$

$\alpha = 0$

$\alpha = 0.2$

$\alpha = 1$

$\alpha = 2$

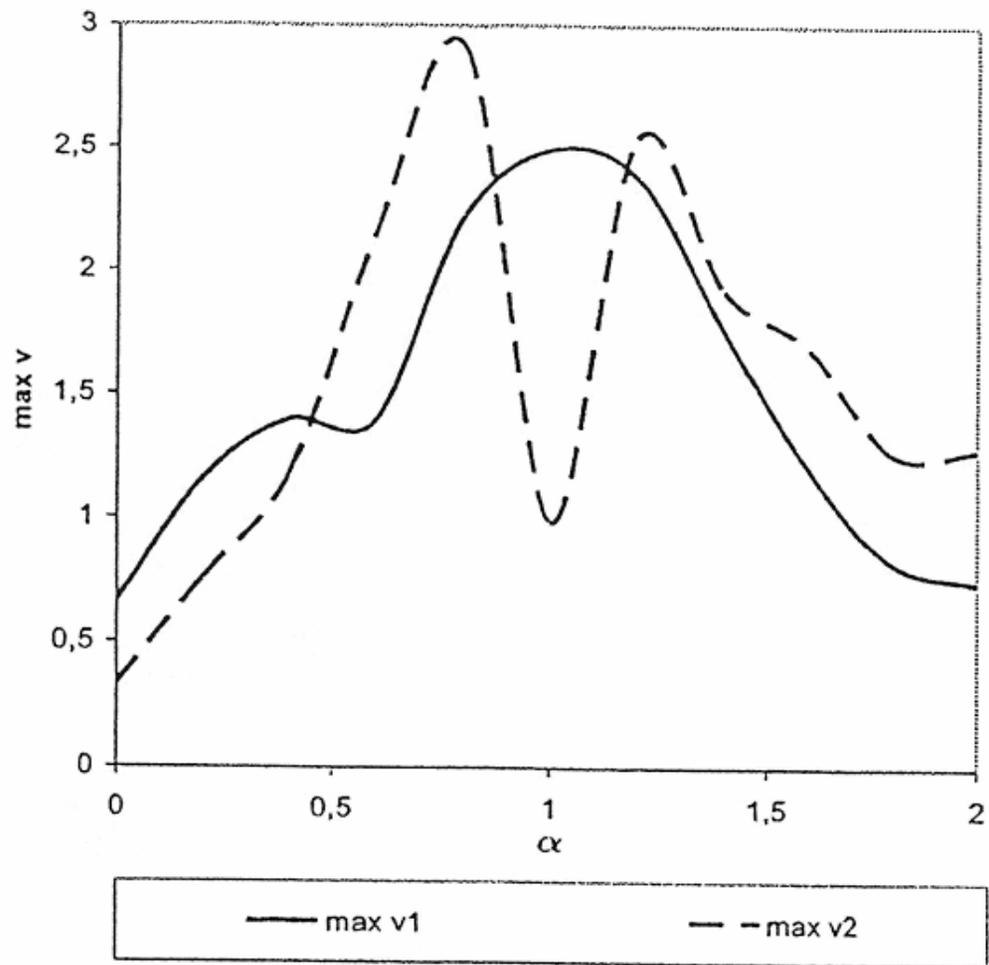


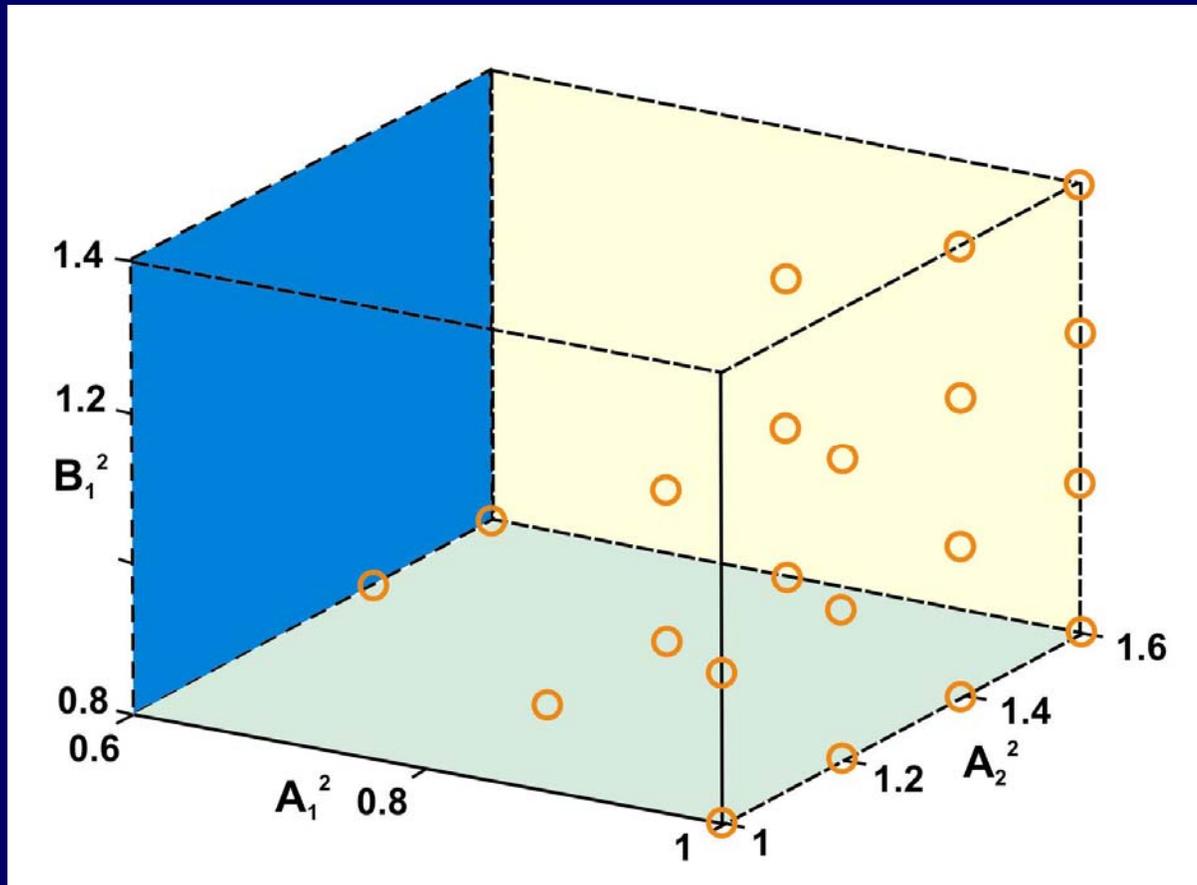
— beam deflection

..... string deflection

$\alpha$  = dimensionless speed

Effect of c





At  $\alpha = 0$ :

$v_1(x) < 1$  for  $B_1^2 < A_1^2 A_2^2$

$v_2(x) < 1$  for  $B_2^2 < A_1^2 A_2^2$

## 5. Conclusions

- Dynamic effects on bridges rise with increasing speeds of trains
- The dynamic response of concrete bridges is a little lower than steel ones due to their different mass and damping ratios
- Stress spectra present important data for the assessment of bridges at fatigue
- Vertical accelerations of bridges may be a limit state for the design of new high speed lines
- The diminishing of bridge dynamic response could be achieved with dampers or by interconnection of prestressed beams with pretensioned strings

