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Dynamic excitations of transport structures

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Outline

- Introduction
- Movement of loads
- Rails
- Random vibration of structures
- Degradation of structural materials under dynamic loads
- Conclusions

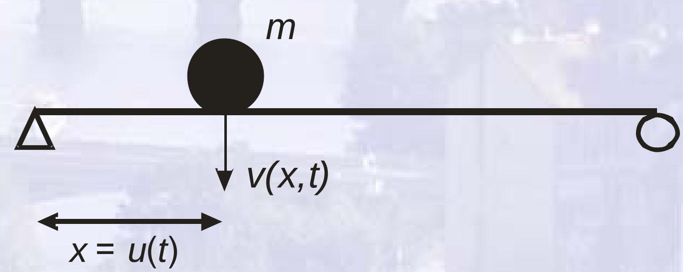
Introduction

Transport structures – moving load
– earthquake
– wind

Movement of the load

Moving load

$$f(x,t) = \delta\left[x - u(t)\right]\left\{F - m\frac{d^2 v\left[u(t),t\right]}{dt^2}\right\}$$



$$\frac{d^2 v\left[u(t),t\right]}{dt^2} = \frac{\partial^2 v}{\partial u^2}\left(\frac{du}{dt}\right)^2 + 2\frac{\partial^2 v}{\partial u\, \partial t}\frac{du}{dt} + \frac{\partial v}{\partial u}\frac{d^2 u}{dt^2} + \frac{\partial^2 v}{\partial t^2}$$

Uniform movement at velocity *c*

$$x = u(t) = ct, \quad \frac{du}{dt} = c \quad \frac{d^2u}{dt^2} = 0$$

$$\frac{d^2 v(ct,t)}{dt^2} = \left[\frac{\partial^2 v(x,t)}{\partial t^2} + 2c \frac{\partial^2 v(x,t)}{\partial x\, \partial t} + c^2 \frac{\partial^2 v(x,t)}{\partial x^2} \right]_{x=ct}$$

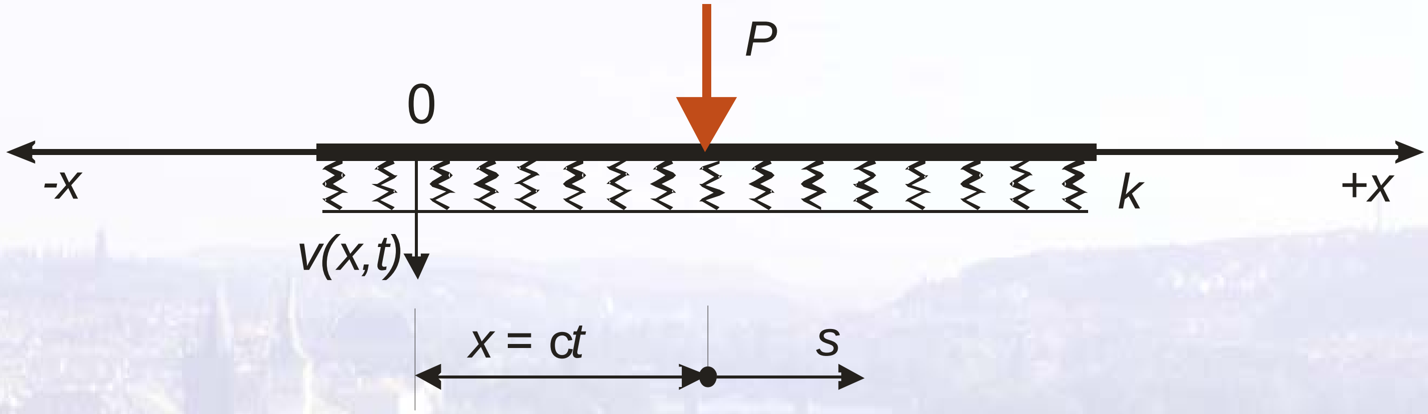
Motion in (x,y) plane $x = u(t)$, $y = v(t)$

load $f(x,y,t) - m(x,y,t)\, d^2w / dt^2$

$$\frac{d^2 w}{dt^2} = \left[\frac{\partial^2 w}{\partial x^2} \left(\frac{du}{dt} \right)^2 + \frac{\partial^2 w}{\partial y^2} \left(\frac{dv}{dt} \right)^2 + \frac{\partial^2 w}{\partial t^2} + 2 \frac{\partial^2 w}{\partial x\, \partial y} \frac{du}{dt} \frac{dv}{dt} + \right.$$

$$\left. + 2 \frac{\partial^2 w}{\partial x\, \partial t} \frac{du}{dt} + 2 \frac{\partial^2 w}{\partial y\, \partial t} \frac{dv}{dt} + \frac{\partial w}{\partial x} \frac{d^2 u}{dt^2} + \frac{\partial w}{\partial y} \frac{d^2 v}{dt^2} \right]_{\substack{x=u(t) \\ y=v(t)}}$$

Rails



Diferencial equation

$$EIv^{IV}(x,t) + \mu\ddot{v}(x,t) + 2\mu\omega_b\dot{v}(x,t) + kv(x,t) = \delta(x-ct)P$$

Steady state vibration

$$v(x,t) = v_0 v(s), \qquad v_0 = \frac{P}{8\lambda^3 EI} = \frac{P\lambda}{2k}$$

$$\lambda = \left(\frac{k}{4EI}\right)^{1/4}$$

Moving coordinate: $s = \lambda(x - ct)$

Speed parameter: $$\alpha = \frac{c}{c_{cr}} = \frac{c}{2\lambda}\left(\frac{\mu}{EI}\right)^{1/2}$$

Damping parameter: $\beta = \omega_b(\mu / k)^{1/2}$

Critical speed: $$c_{cr} = 2\lambda\left(\frac{EI}{\mu}\right)^{1/2}$$

Ordinary differential equation:

$$v^{IV}(s) + 4\alpha^2 v^{II}(s) - 8\alpha\beta v^{I}(s) + 4v(s) = 8\delta(s)$$

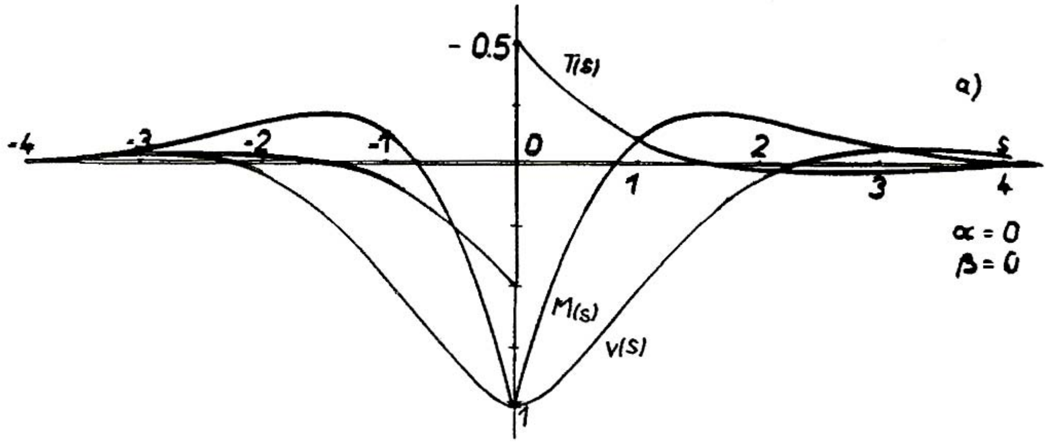
Solution

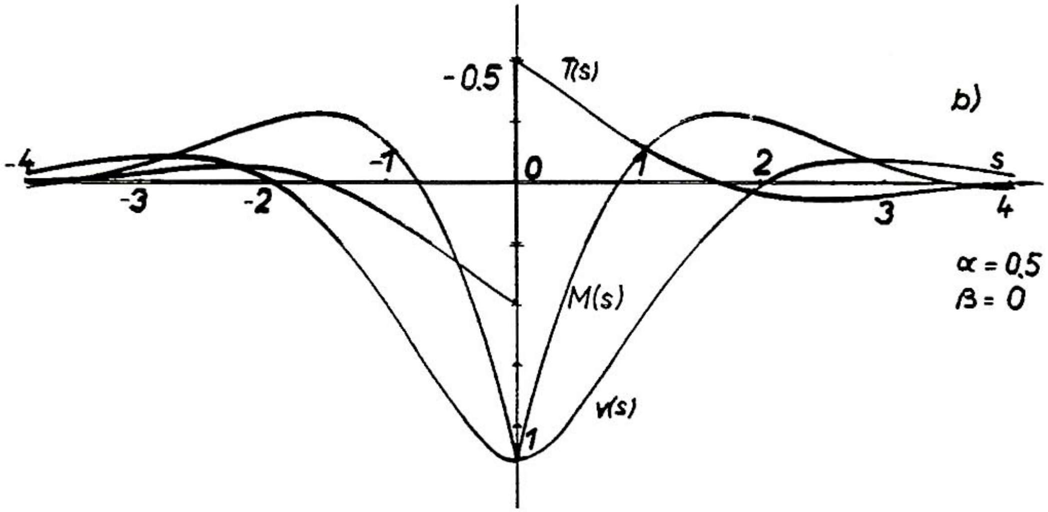
$$v(s) = \frac{2}{a_1(D_1^2 + D_2^2)} e^{-bs} (D_1 \cos \; a_1 s + D_2 \sin \; a_1 s)$$

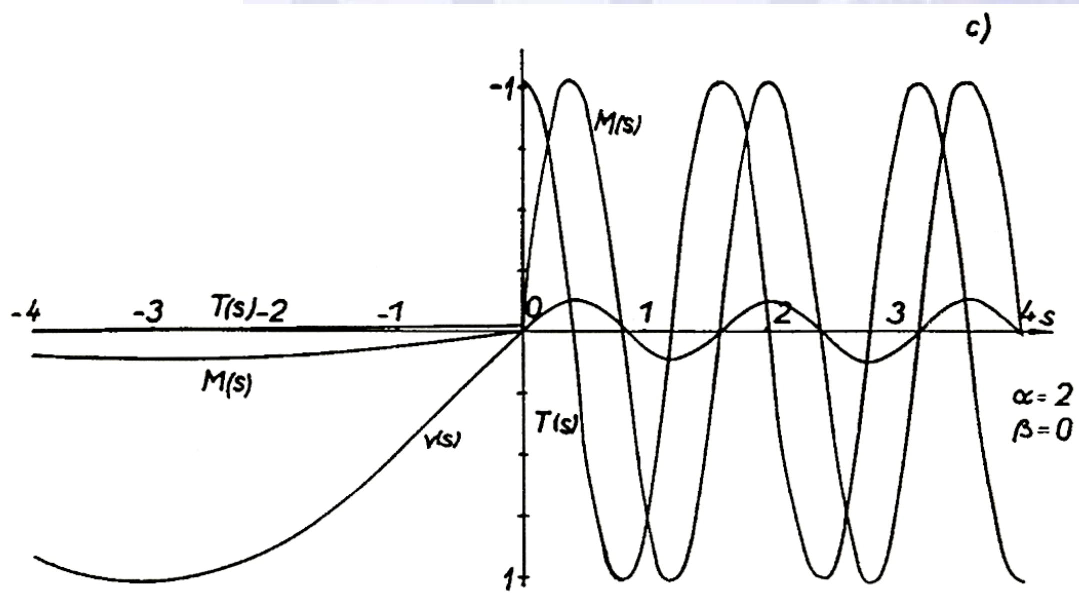
$$\text{for } \; s > 0$$

$$v(s) = \frac{2}{a_2(D_3^2 + D_4^2)} e^{bs} (D_3 \cos \; a_2 s + D_4 \sin \; a_2 s)$$

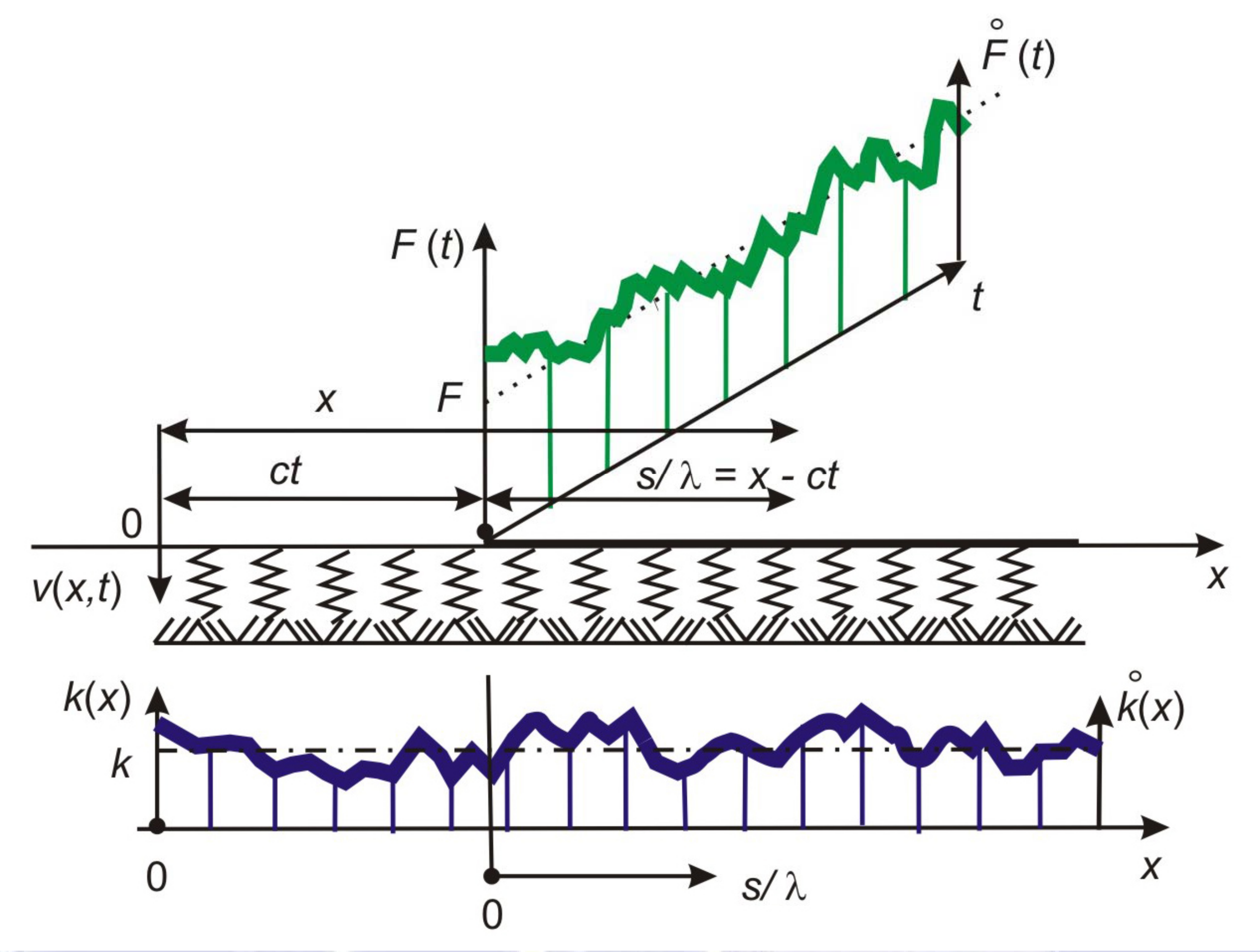
$$\text{for } \; s < 0$$


a)
- 0.5
T(s)
M(s)
v(s)
-4
-3
-2
-1
0
1
2
3
4
s
1
α = 0
β = 0


b)
- 0.5
T(s)
M(s)
v(s)
-4
-3
-2
-1
0
1
2
3
4
s
1
α = 0,5
β = 0


c)
-1
M(s)
-4
-3
T(s)-2
-1
0
1
2
3
4
s
M(s)
v(s)
T(s)
1
α = 2
β = 0

Random vibration of a beam on elestic foundation under moving force



Equation of the beam

$$L\left[v(x,t)\right] \equiv EIv^{IV}(x,t) + \mu\ddot{v}(x,t) + 2\mu\omega_b\dot{v}(x,t) + k(x)v(x,t) =$$

$$= \delta(x-ct)\ F(t)$$

Foundation

$$k(x) = k + \varepsilon\,\mathring{k}(x)$$

$$k = E\left[k(x)\right]$$

Force

$$F(t) = F + \varepsilon\,\mathring{F}(t)$$

$$F = E\left[F(t)\right]$$

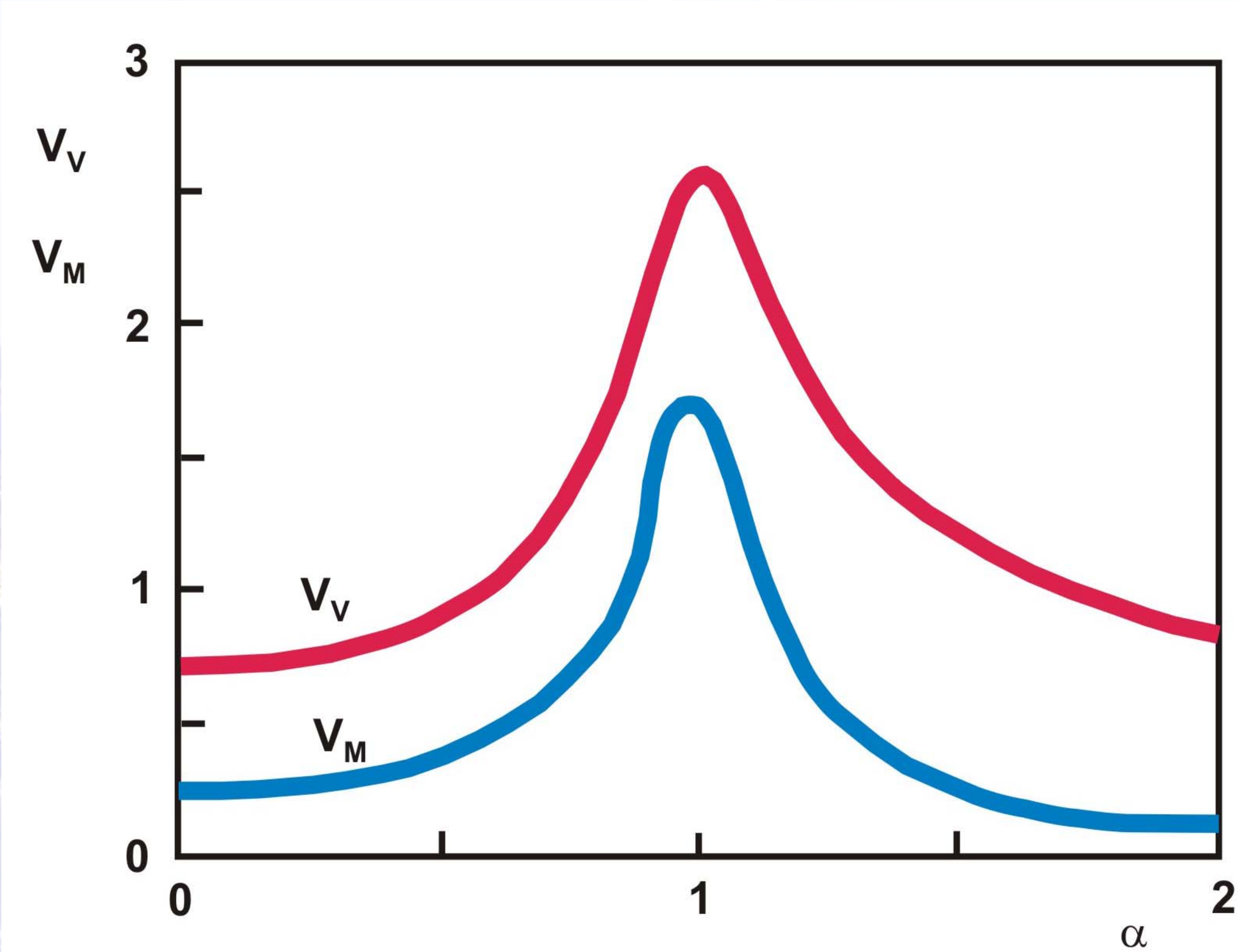
$$\varepsilon \ll 1$$

Moving coordinate

$$s = \lambda(x-ct)\ ,\quad \lambda = \left(\frac{k}{4EI}\right)^{1/4}$$

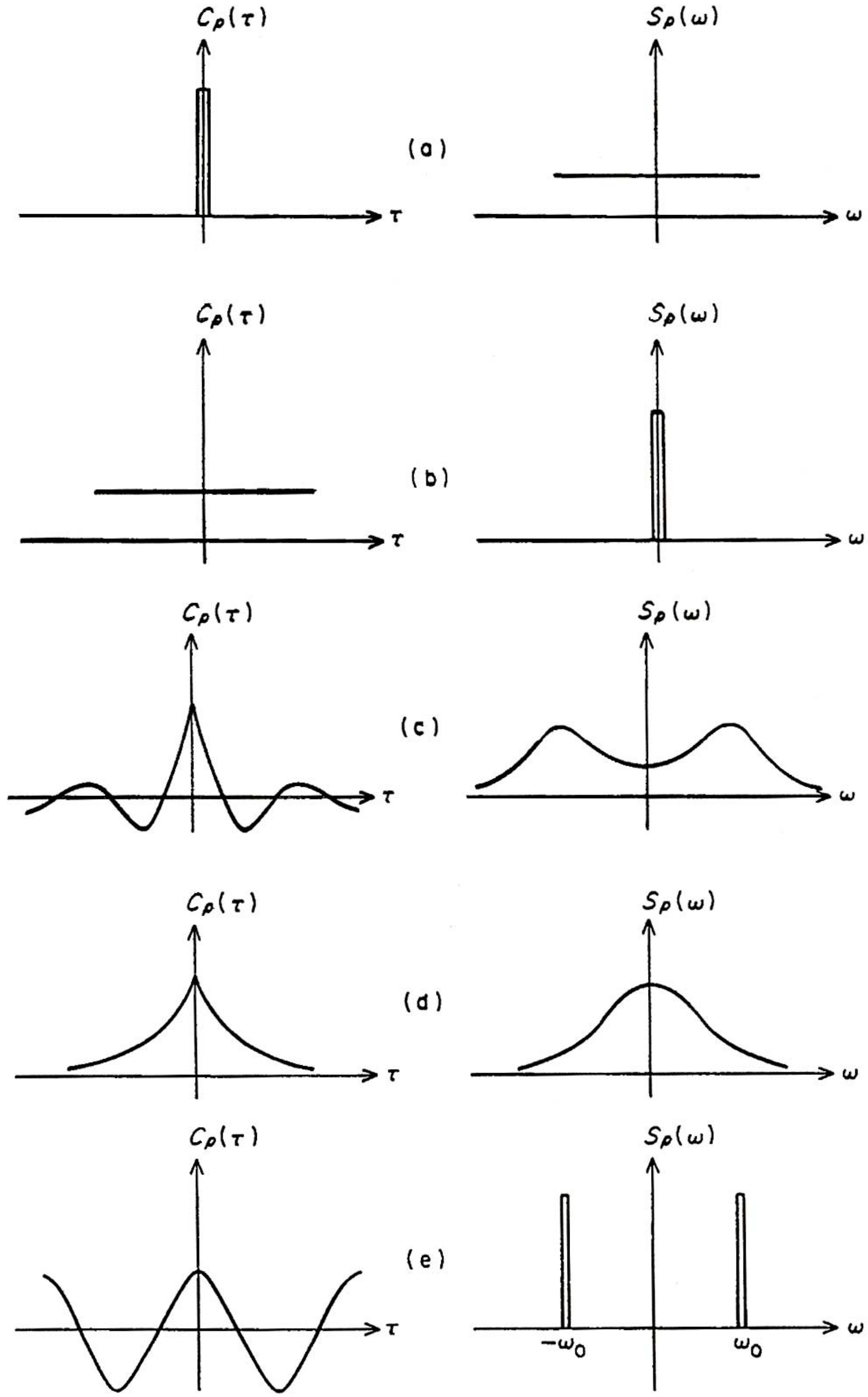
Static deflection and bending moment under force *F*

$$v_0 = \frac{F}{8\lambda^3 EI} = \frac{F\lambda}{2k}\ ,\quad M_0 = \frac{F}{4\lambda}$$



Coeficient of variation V_V and V_M as a function of speed α,
$\beta = 0.2$, $\gamma_i = 0$, $\sigma = 0.2$, D = $10/\lambda$

COVARIANCE



POWER SPECTRAL DENSITY

Random vibration of a beam

Bernoulli-Euler equation

$$EI\ v^{IV}(x,t) + \mu\ddot{v}(x,t) + 2\mu\omega_b\dot{v}(x,t) = p(x,t)$$

Normal mode analysis

$$v(x,t) = \sum_{j=1}^{\infty} v_j(x)\ q_j(t)$$

$$p(x,t) = \sum_{j=1}^{\infty} \mu\ v_j(x)\ Q_j(t)$$

Generalized deflection

$$\ddot{q}_j(t) + 2\omega_b\dot{q}_j(t) + \omega_j^2\ q_j(t) = Q_j(t)$$

Generalized force

$$Q_j(t) = \begin{cases} \dfrac{1}{M_j}\displaystyle\int_0^L p(x,t)\ v_j(x)\ dx \\ 0 \end{cases} \quad \text{for } t \begin{cases} \geq 0 \\ < 0 \end{cases}$$

Solution

$$q_j(t) = \int_{-\infty}^{\infty} h_j(t-\tau)\ Q_j(\tau)d\tau = \int_{-\infty}^{\infty} h_j(\tau)\ Q_j(t-\tau)d\tau$$

Impulse function (weighting function)

$$h_j(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_j(\omega)e^{i\tau\omega}d\omega$$

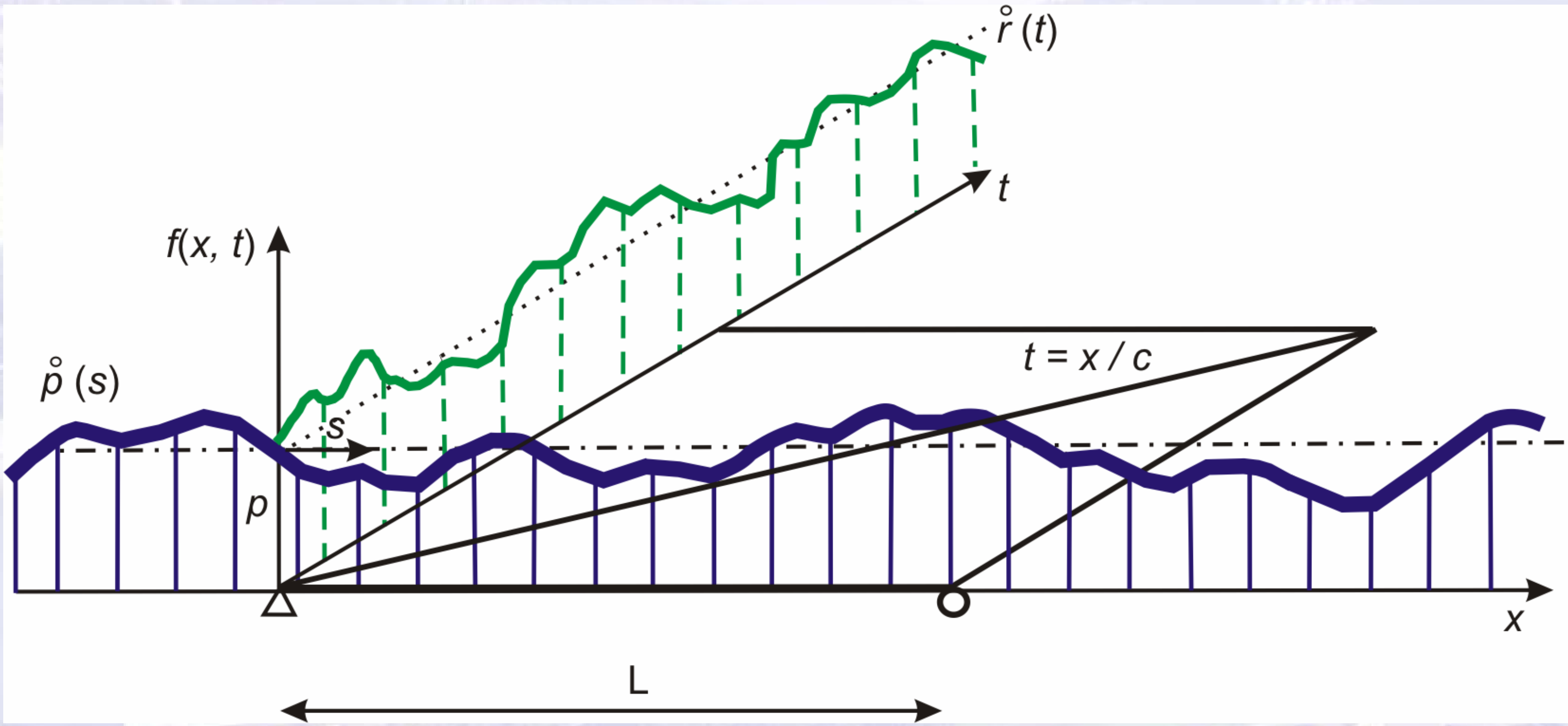
Frequency response function

$$H_j(\omega) = \int_{-\infty}^{\infty} h_j(\tau)e^{-i\omega\tau}d\tau$$

Load

$$p(x,t) = E\left[p(x,t)\right] + \overset{\circ}{p}(x,t)$$

Stationary vibration of a beam under moving continuous random load



Load

$$p(x,t)=\left[\,p+\varepsilon\,\mathring{p}(s)\right]\cdot\left[1+\varepsilon\,\mathring{r}(t)\right]$$

$$p=E\left[p(x,t)\right]$$

$$\mathring{p}(x,t)=\varepsilon\,\mathring{p}(s)+\varepsilon\,p\,\mathring{r}(t)+\varepsilon^{2}\,\mathring{p}(s)\,\mathring{r}(t)$$

$$s=t-x/c$$

$$\varepsilon \ll 1$$

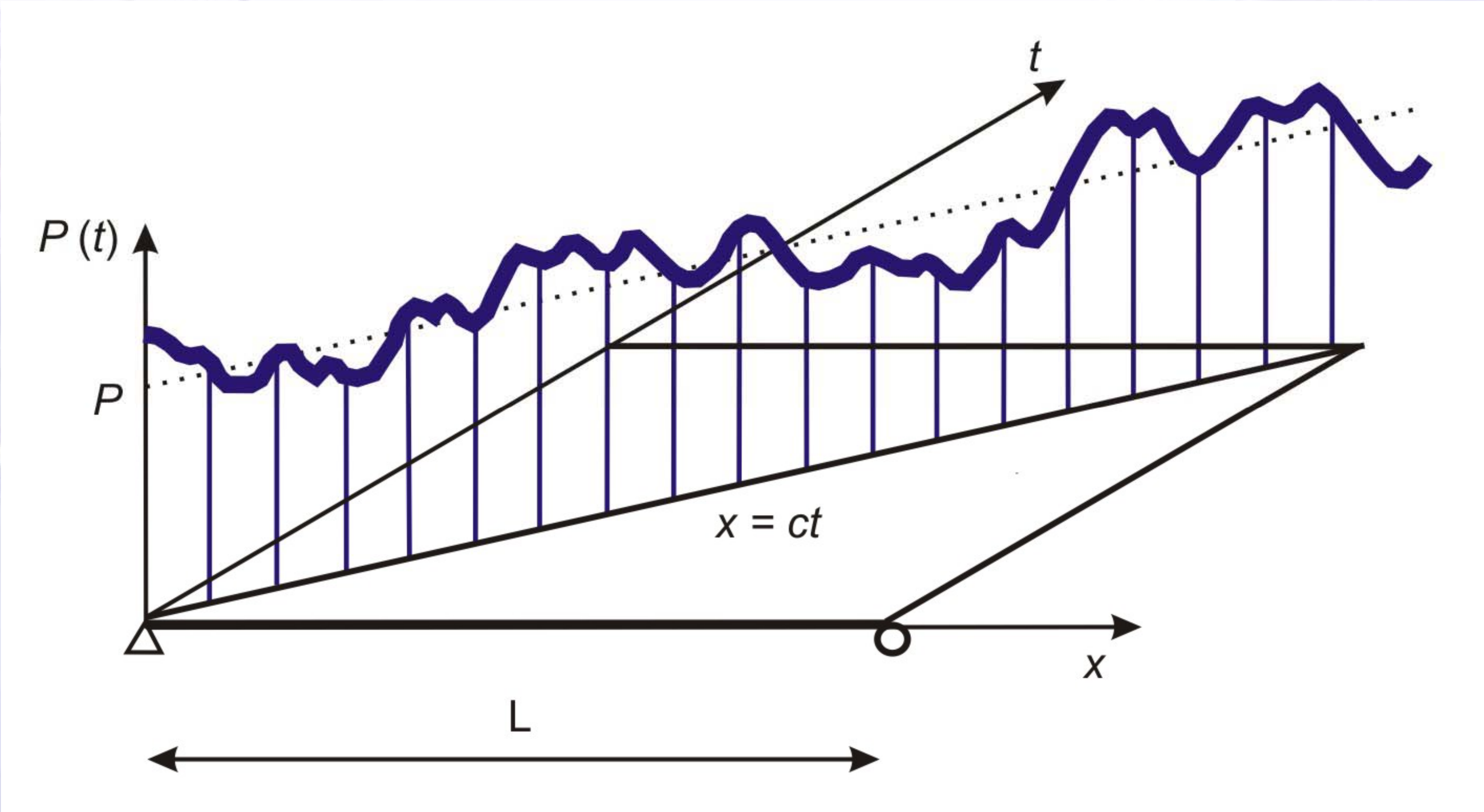
Covariance of the moving load

$$C_{pp}(x_1,x_2,\tau)=\varepsilon^{2}C_{pp}(\tau+\tau_0)+\varepsilon^{2}p^{2}C_{rr}(\tau)+\varepsilon^{2}pC_{pr}(\tau+x_1/c)+$$

$$+\varepsilon^{2}pC_{rp}(\tau-x_2/c)+\varepsilon^{3}C_{ppr}+\varepsilon^{3}pC_{prr}+\varepsilon^{3}C_{ppr}+\varepsilon^{3}pC_{prr}+\varepsilon^{4}C_{pprr}$$

$$\tau_0=\frac{x_1-x_2}{c}$$

Non-stationary vibration of a beam under moving random force



Load

$$p(x,t)=\delta(x-ct)\ P(t)\ ,\quad P(t)=P+\overset{\circ}{P}(t)\ ,\quad E\left[P(t)\right]=P$$

Covariance of the force

$$C_{pp}(t_1,t_2)=E\left[\overset{\circ}{P}(t_1)\,\overset{\circ}{P}(t_2)\right]$$

Covariance of moving load

$$C_{pp}(x_1,x_2,t_1,t_2)=\delta(x_1-ct_1)\delta(x_2-ct_2)C_{pp}(t_1,t_2)$$

Covariance of generalized deflection

$$C_{q_jq_k}(t_1,t_2)=\frac{1}{M_jM_k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h_j(t_1-\tau_1)h_k(t_2-\tau_2)v_j(c\tau_1)v_k(c\tau_2)C_{pp}(\tau_1,\tau_2)d\tau_1d\tau_2$$

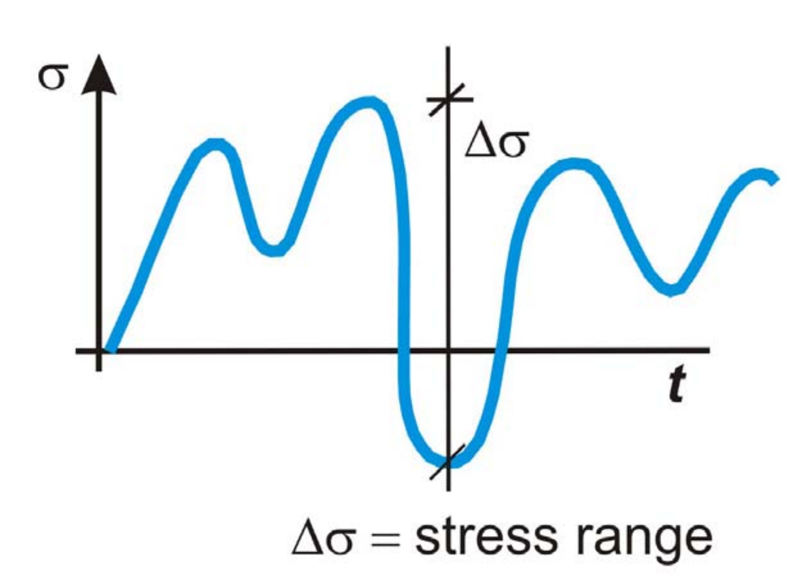
Speed parameter $\alpha=c/(2f_1l)$

Damping parameter $\beta=\omega_b/\omega_1=\vartheta/(2\pi)$

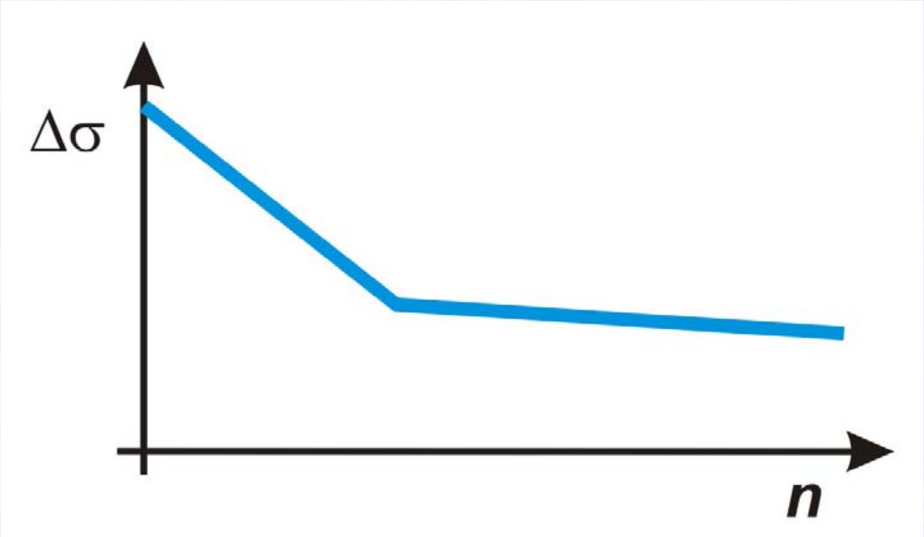
Simple beam $v_j(x)=\sin\dfrac{j\pi x}{l}\ ,\quad v_0=\dfrac{Pl^3}{48EI}$

Degradation of structural materials under dynamic loads

Dynamic loads

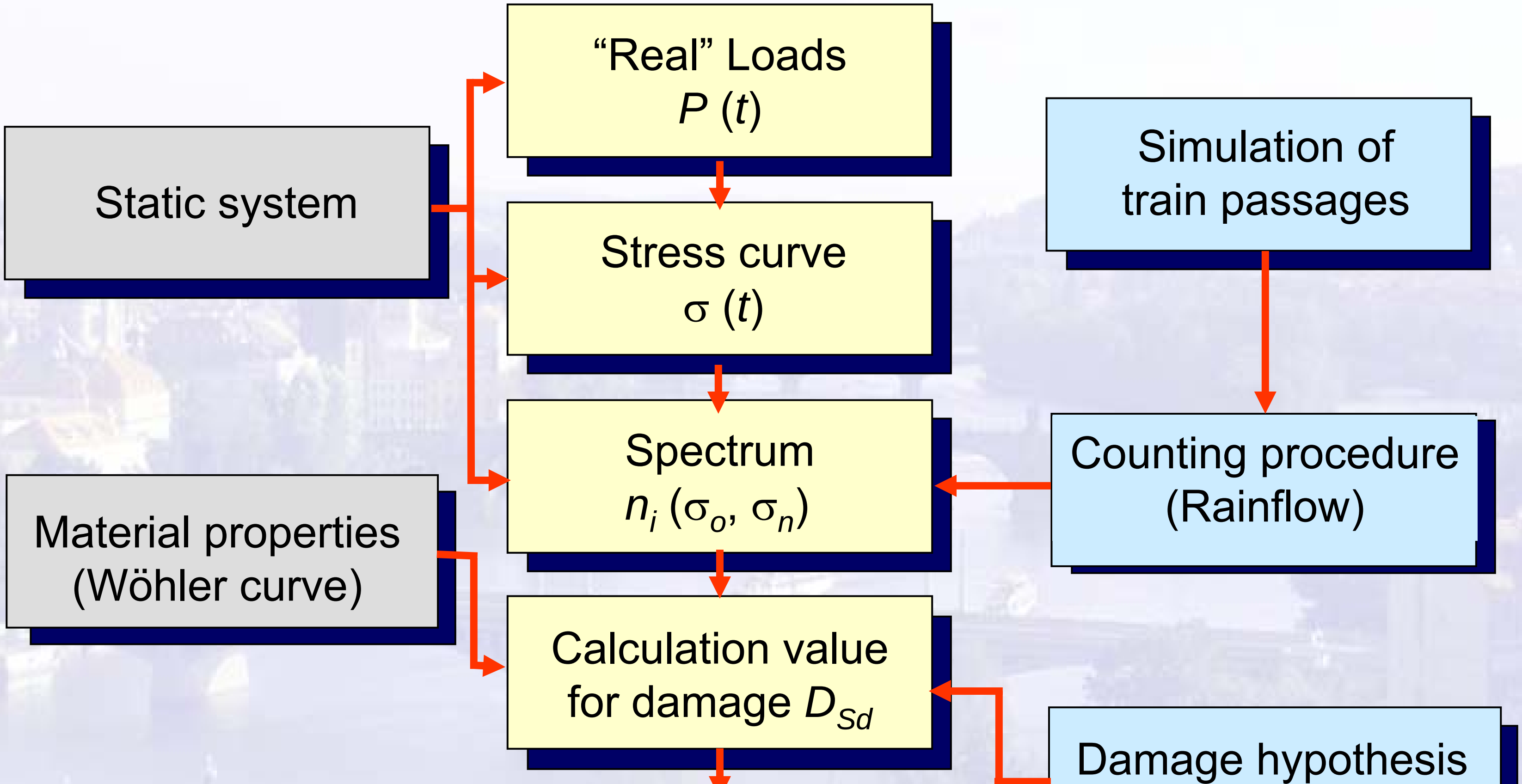


Wöhler curve



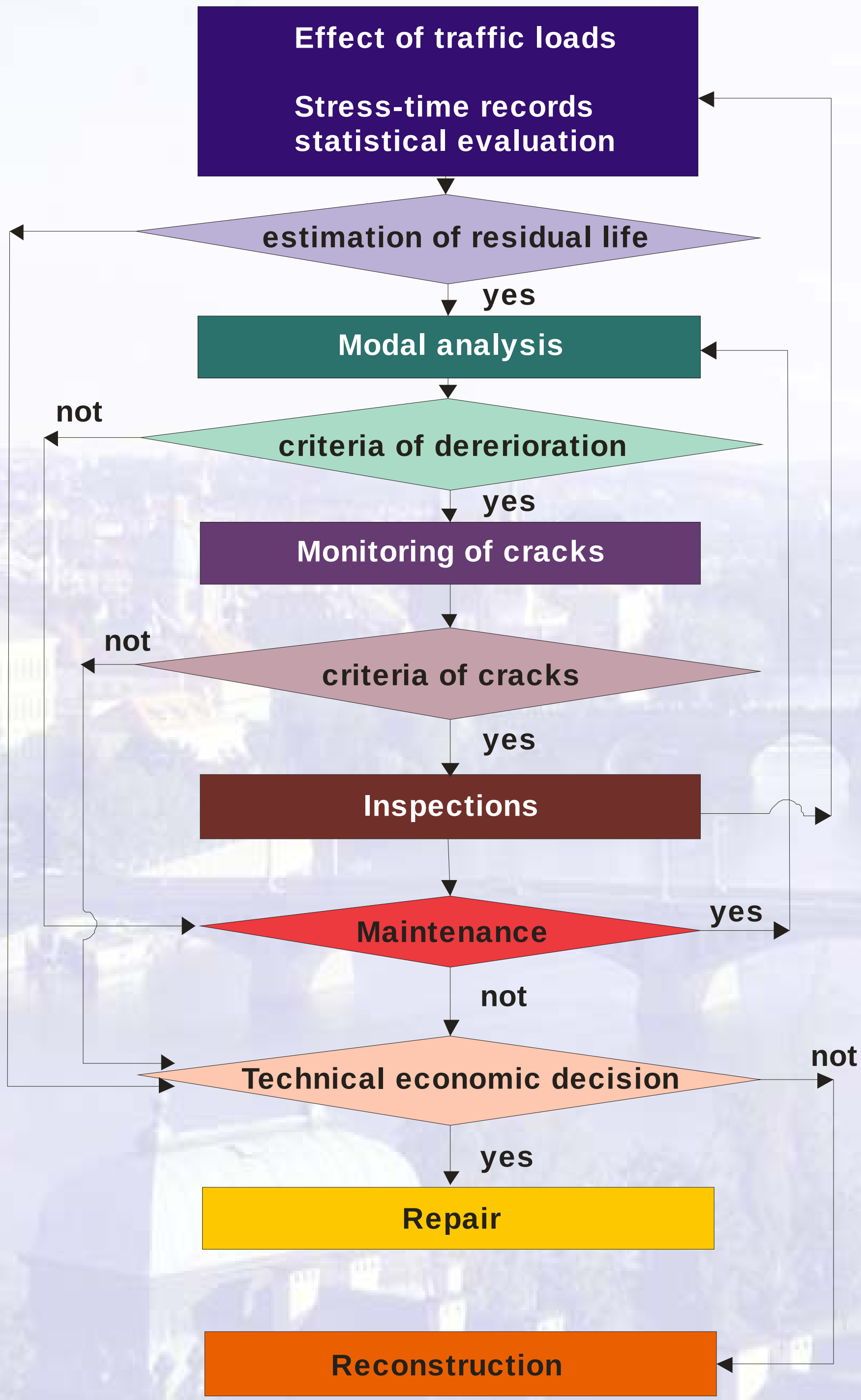
Fatigue tests

Concept of fatigue assessment



$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$\rho = \sigma_{min} / \sigma_{max}$$



Strategy of maintenance, repairs and reconstructions,

yes – the conditions are fulfilled,
not – the conditions are not fulfilled

Conclusions

- **The dynamic loads, i.e. in time varying loads, increase the stresses in transport structures with increasing speed**
- **The dynamic loads are varying in time either regularly or randomly**
- **The response of structural materials deteriorates their properties in time**